Model Robustness in the Equity to Credit Universe

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Equity to Credit:
The Traditional Approach

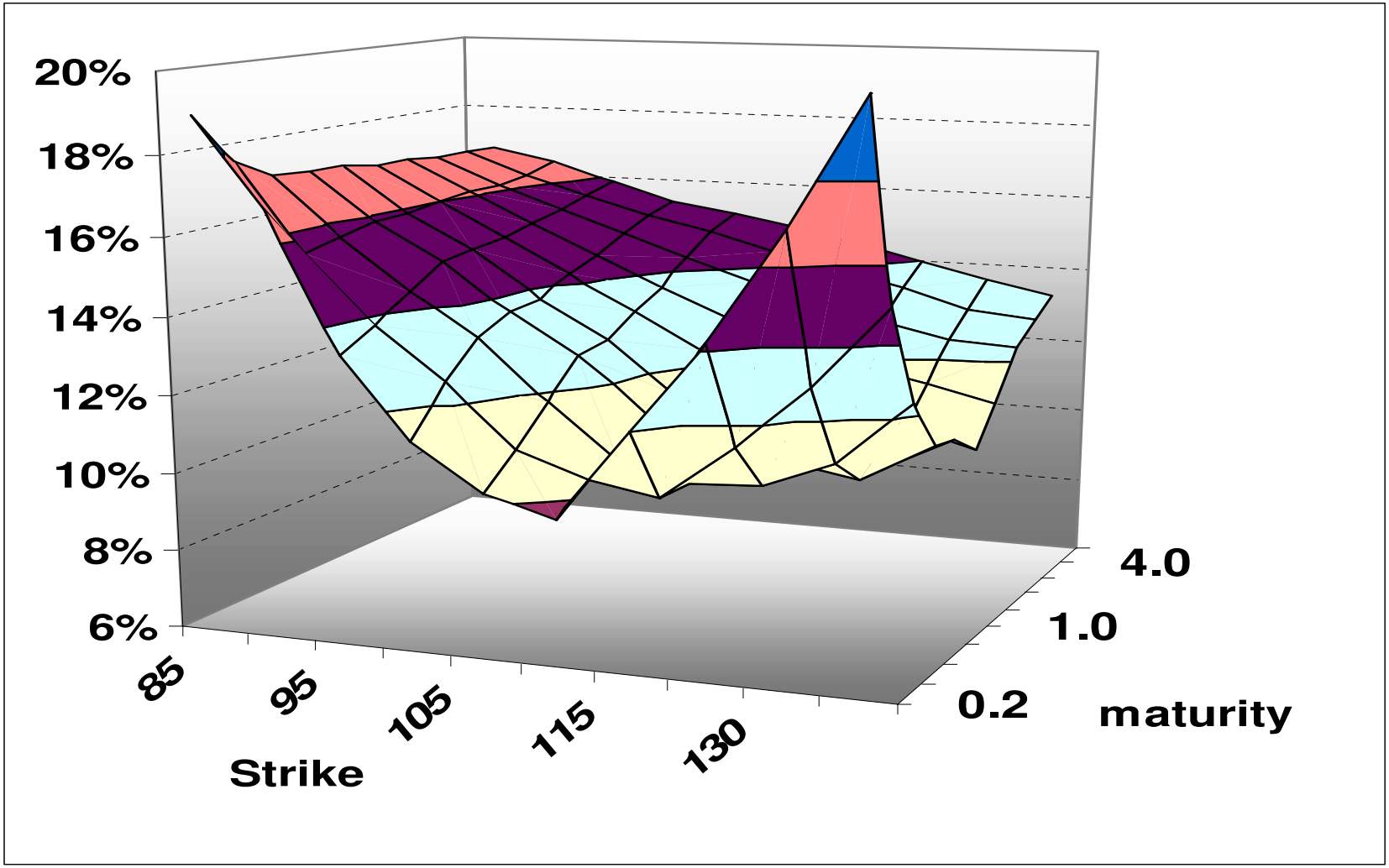
The Traditional Approach

- From local volatility…
- …to local hazard rate

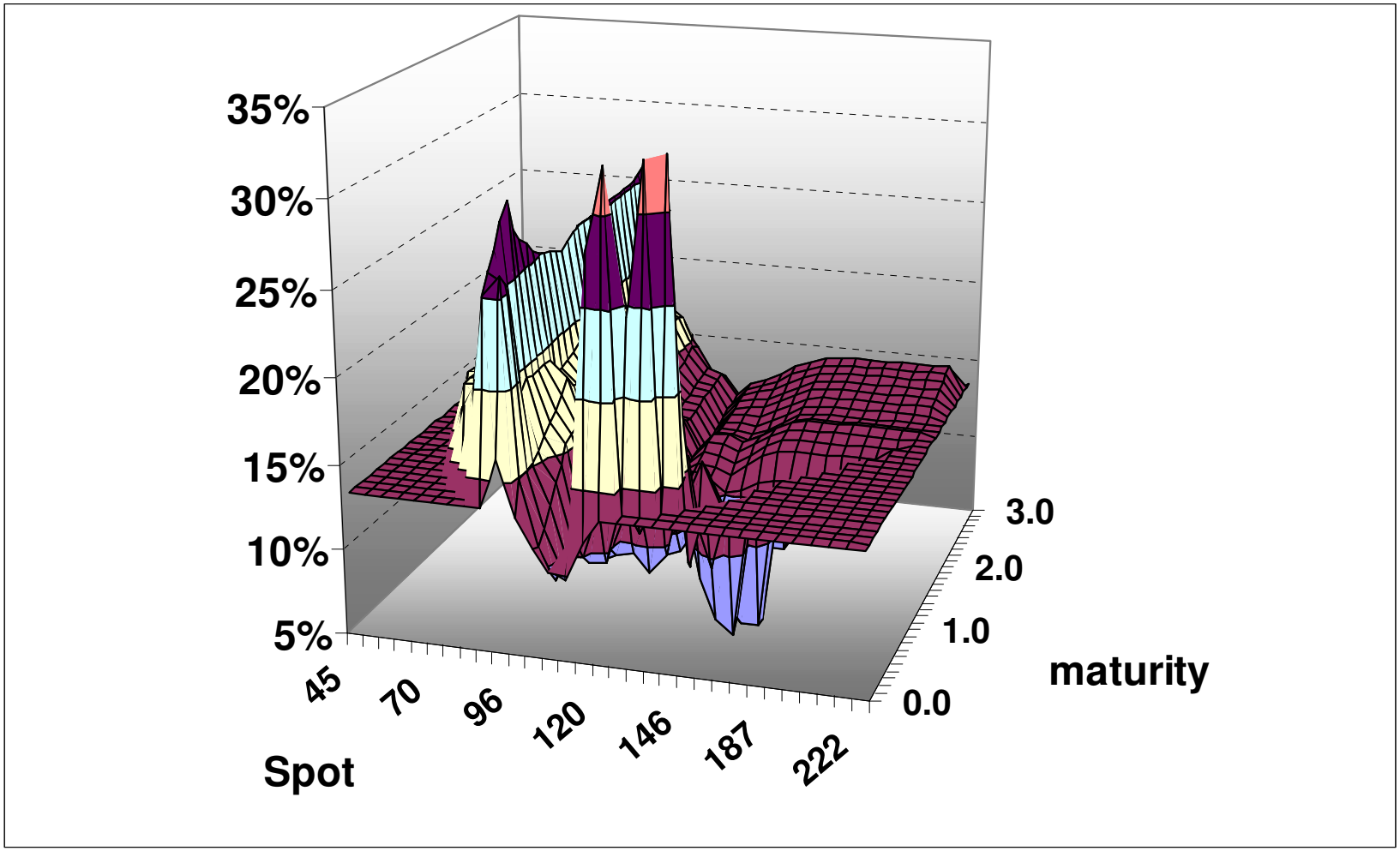
Local Volatility

- The volatility parameter is a function of time and spot: $\sigma(S, t)$
- Natural extension from Black-Scholes analysis
- Defines a pricing kernel (no arbitrage)
- Consistent with a perfect dynamic hedge
- Local vol can be recovered from the smile (Dupire's formula)

S&P 500 Index Smile October 1995



S&P 500 Oct 1995 Local Volatility



Local Volatility

- Numerically ill posed
- Highly unstable
- Prices and hedges of exotics are out of line with the market
- No dynamic hedge with instruments other than the underlying
- Implies a (unique) strange dynamics for future smiles

Local Hazard Rate

- Structural models of the firm: default is triggered by a bankruptcy threshold (Merton, KMV, CreditGrades)
- Reduced-form model: default is triggered by a Poisson process of given intensity (hazard rate)
- Synthesis: the hazard rate is a function of the underlying equity price (and time)

Local Default Intensity

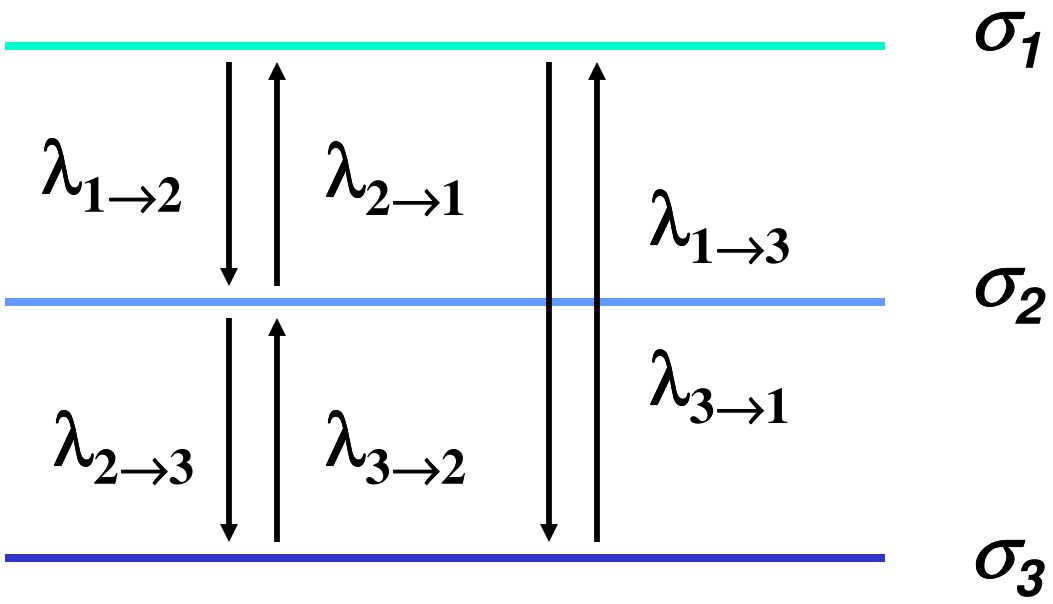
- Generalizes the local vol $\sigma(S, t)$ to the default intensity $p(S, t)$
- Hard to calibrate in general
- In practice simple parametric forms are used
- Lacks robustness
- The stock price – default link should be probabilistic and not deterministic
- The vol – default link should likewise not be deterministic

Advanced Smile Analysis

Advanced Smile Analysis

- Stochastic vol for the long term
- Jumps for the short term
- Correlation vol-stock for the skew
- As few tweaks as possible for robustness (time and spot homogeneity)
- Calibration on some key exotic is essential to pinpoint salient features of the smile dynamics

Nobody's Regimes



- Continuous time Markov transitions
- Underlying may jump between regimes
- Time and spot homogeneity

Nobody's Parameters

	Brownian Diffusion	Total Volatility
Regime 1	9.6%	11.7%
Regime 2	6.2%	32.3%
Regime 3	2.3%	11.7%

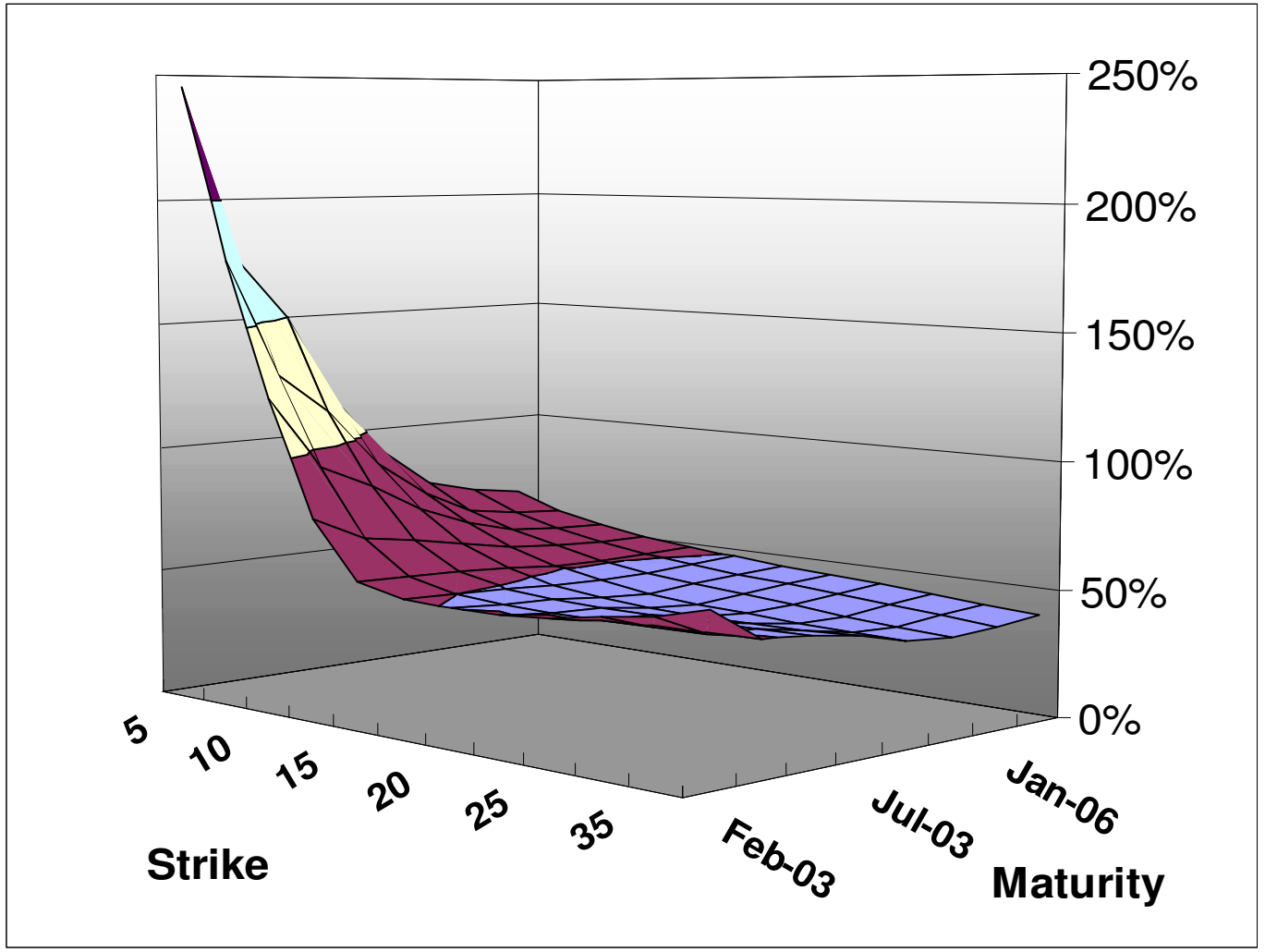
	Jump Size	Jump Intensity
Regime 1 -> Regime 2	-9.1%	0.24
Regime 2 -> Regime 1	62.7%	0.09
Regime 1 -> Regime 3	2.7%	3.40
Regime 3 -> Regime 1	-3.2%	2.98
Regime 2 -> Regime 3	24.6%	1.09
Regime 3 -> Regime 2	-22.7%	0.20

Advanced Credit Analysis

E-to-C Analysis

- Default is an extreme case of jump
- Which causes the CB bond floor to collapse (negative gamma)
- Historic vol misses default completely
- Total vol = diffusion vol + jump
- But it can be recovered from the term structure of CDS premia …
- …and the smile (creates a skew)

Tyco Smile 3 February 2003



Advanced Credit Analysis

- Credit is more than a default event
- Credit is stochastic
- Structural models ignore significant corporate events
- M&A transaction?
- LBO?
- Restructuring?
- All these are stochastic events
- Everything depends on S is dangerously naive

Equity to Credit

Spot cannot be the only state variable

- Spot
- Credit spread
- Default status
- Volatility
- Corporate events

Equity to Credit

We must price in a consistent way:

- Vanilla and exotic options
- CDS and options on CDS
- Equity Default Swaps …

- …together with the Convertible Bonds (and options written on it)

The Convertible Bond Nightmare

- CB are totally main stream …
- … and totally exotic (soft calls, triggers)
- They are also credit instruments
- Traders face one big question:

Which implied volatility should they plug in their CB pricer?

Equity to Credit: Incomplete Markets

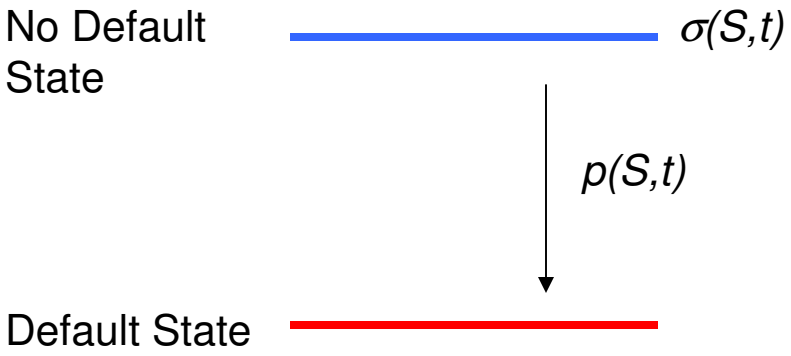
Incomplete Markets

- Large state space implies incompleteness
- Several pricing kernels are possible
- Usually no perfect hedge
- But we keep from Black-Scholes the notion of self financing portfolios
- Price derived from cost of optimal hedge
- It becomes meaningful to hedge with several instruments
- and to measure the hedge quality: HERO

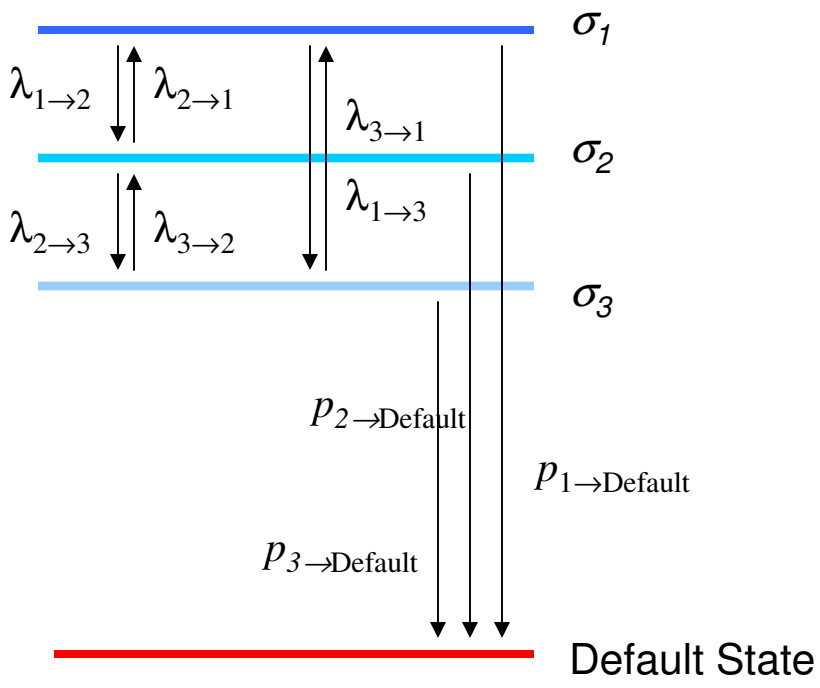
Incomplete markets

- The state of no-default decomposes into sub-regimes of different diffusion components and different hazard rates
- This replaces $\sigma(S, t)$ and $p(S, t)$ with stochastic σ and stochastic p
- It turns the model into a homogenous model
- Markov transition matrix between regimes
- Stock jumps between regimes yield the needed stock – vol – default correlations

Inhomogeneous



Homogeneous



Some Regime Experiments

Pure Black-Schloles

- Vol 40%
- No default

Maturity	Spread
1	0.00%
2	0.00%
3	0.00%
5	0.00%
10	0.00%

	Strike											
Maturity	5.00	7.50	10.00	12.50	14.00	15.00	16.00	17.50	20.00	25.00	30.00	40.00
1/3/05				40.38%	39.96%	39.92%	39.94%	40.21%				
14/3/05			40.55%	40.03%	39.99%	39.97%	39.98%	40.02%				
14/5/05		40.09%	40.00%	40.00%	40.00%	40.00%	40.00%	40.00%	40.00%	40.02%		
14/8/05	40.12%	40.00%	40.00%	40.00%	40.00%	40.00%	40.00%	40.00%	40.00%	40.00%	40.00%	
14/2/06	40.00%	40.00%	40.00%	40.00%	40.00%	40.00%	40.00%	40.00%	40.00%	40.00%	40.00%	40.00%
14/2/07	40.00%	40.00%	40.00%	40.00%	40.00%	40.00%	40.00%	40.00%	40.00%	40.00%	40.00%	40.00%
14/2/08	40.00%	40.00%	40.00%	40.00%	40.00%	40.00%	40.00%	40.00%	40.00%	40.00%	40.00%	40.00%
14/2/10	40.00%	40.00%	40.00%	40.00%	40.00%	40.00%	40.00%	40.00%	40.00%	40.00%	40.00%	40.00%

Black-Schloles with positive jump

- Brownian vol 40%
- Jump size 10%
- Intensity 10
- Larger total vol
- Positive skew
- Long term flat

	Strike											
Maturity	5.00	7.50	10.00	12.50	14.00	15.00	16.00	17.50	20.00	25.00	30.00	40.00
1/3/05			48.80%	46.03%	47.79%	49.85%	52.13%	55.51%	60.46%			
14/3/05		52.03%	46.00%	47.72%	49.21%	50.23%	51.24%	52.72%	54.95%	58.68%		
14/5/05	48.26%	47.48%	48.57%	49.56%	50.09%	50.42%	50.73%	51.17%	51.84%	52.97%	53.92%	
14/8/05	48.09%	48.88%	49.52%	50.03%	50.29%	50.46%	50.61%	50.82%	51.15%	51.69%	52.14%	
14/2/06	49.23%	49.68%	50.00%	50.26%	50.39%	50.47%	50.55%	50.66%	50.81%	51.08%	51.30%	51.65%
14/2/07	49.85%	50.08%	50.25%	50.38%	50.44%	50.48%	50.52%	50.57%	50.65%	50.78%	50.89%	51.06%
14/2/08	50.06%	50.22%	50.33%	50.42%	50.46%	50.49%	50.51%	50.55%	50.60%	50.69%	50.76%	50.87%
14/2/10	50.23%	50.33%	50.39%	50.45%	50.47%	50.49%	50.50%	50.52%	50.56%	50.61%	50.65%	50.72%

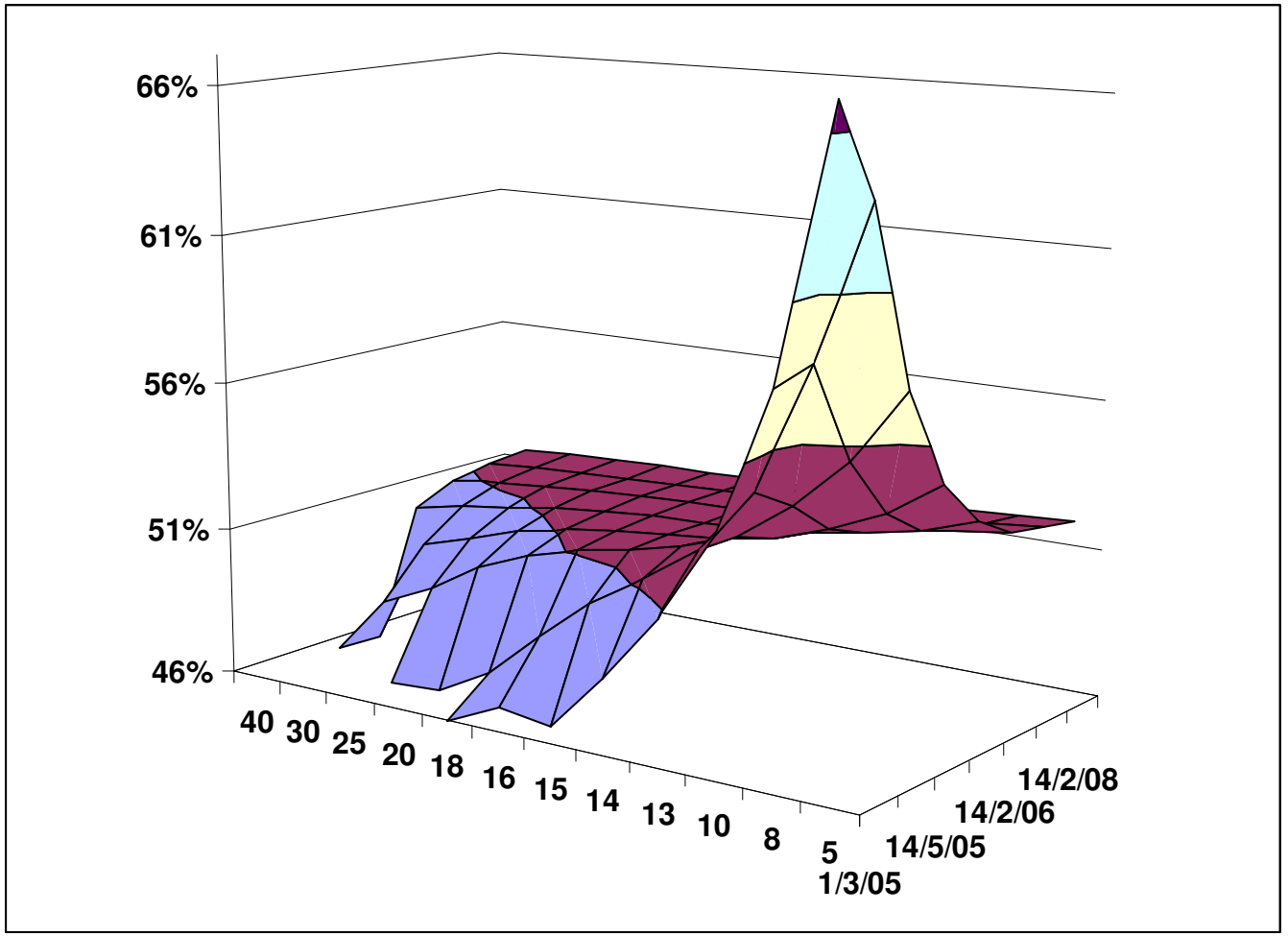
Black-Schloles with negative jump

- Brownian vol 40%
- Jump size -10%
- Intensity 10

- Negative skew

Maturity	Strike											
	5.00	7.50	10.00	12.50	14.00	15.00	16.00	17.50	20.00	25.00	30.00	40.00
1/3/05			66.97%	58.37%	53.63%	50.78%	48.47%	46.55%	46.80%			
14/3/05		63.89%	58.78%	54.63%	52.50%	51.23%	50.11%	48.71%	47.08%	46.10%		
14/5/05	57.88%	55.45%	53.77%	52.49%	51.85%	51.46%	51.10%	50.62%	49.91%	48.81%	48.04%	
14/8/05	54.60%	53.44%	52.63%	52.01%	51.70%	51.51%	51.33%	51.09%	50.73%	50.15%	49.68%	
14/2/06	53.04%	52.48%	52.08%	51.78%	51.62%	51.53%	51.44%	51.32%	51.14%	50.85%	50.60%	50.23%
14/2/07	52.29%	52.01%	51.82%	51.66%	51.59%	51.54%	51.50%	51.44%	51.35%	51.20%	51.07%	50.88%
14/2/08	52.04%	51.86%	51.73%	51.63%	51.58%	51.54%	51.51%	51.47%	51.41%	51.31%	51.23%	51.10%
14/2/10	51.85%	51.74%	51.66%	51.60%	51.57%	51.55%	51.53%	51.51%	51.47%	51.41%	51.36%	51.28%

Black-Schloles with negative jump



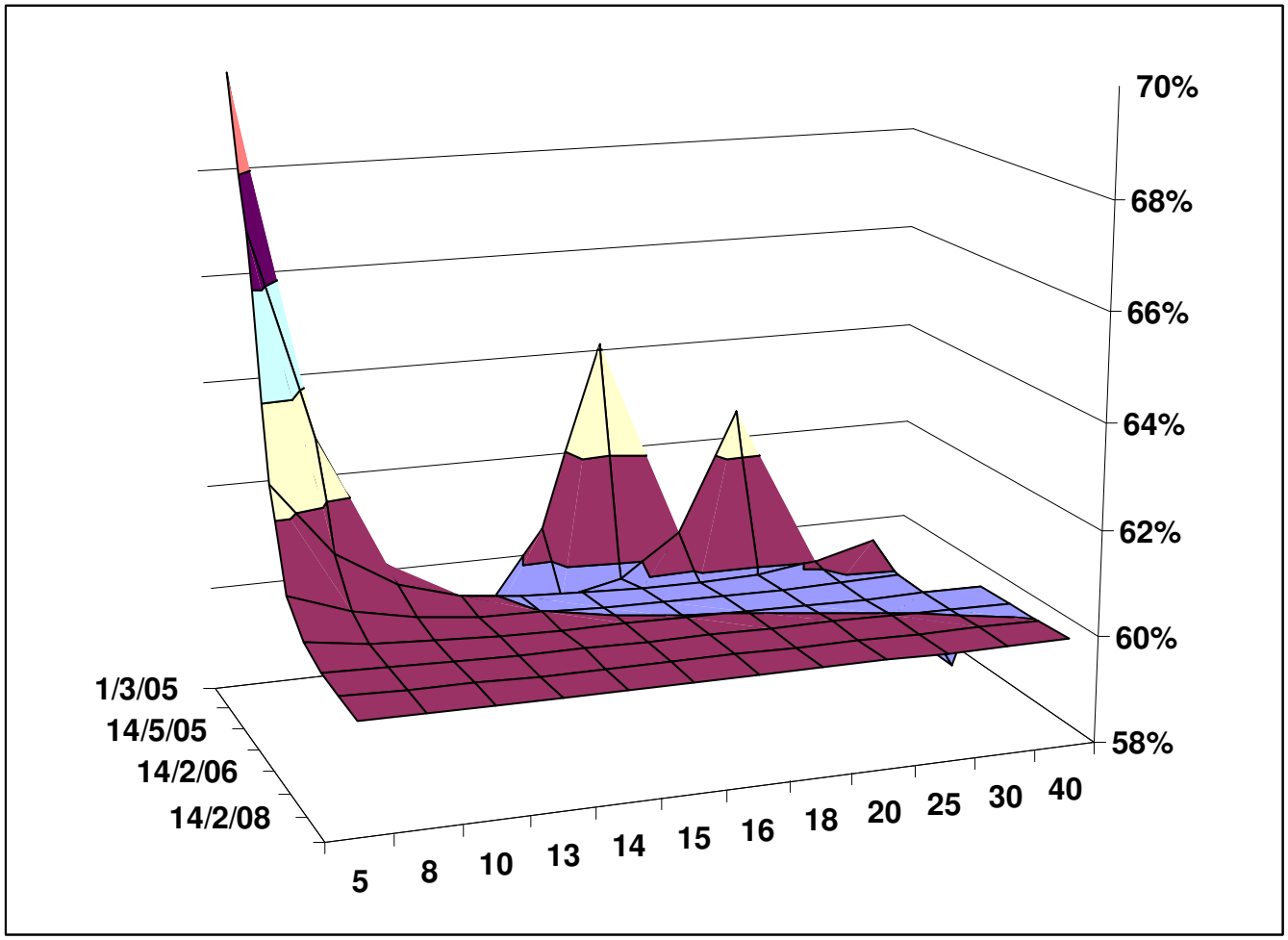
BS with positive and negative jumps

- Brownian vol 40%
- Jump -10%
- Jump 10%
- Intensity 10

- Smile
- Long term flat

Maturity	Strike											
	5.00	7.50	10.00	12.50	14.00	15.00	16.00	17.50	20.00	25.00	30.00	40.00
1/3/05			69.78%	62.54%	59.62%	58.77%	58.93%	60.50%	64.11%			
14/3/05		67.07%	63.06%	60.55%	59.76%	59.50%	59.45%	59.68%	60.52%	62.84%		
14/5/05	62.58%	61.18%	60.46%	60.08%	59.95%	59.89%	59.86%	59.84%	59.86%	60.04%	60.32%	
14/8/05	60.84%	60.41%	60.18%	60.05%	60.00%	59.98%	59.96%	59.94%	59.92%	59.93%	59.96%	
14/2/06	60.32%	60.18%	60.10%	60.05%	60.03%	60.02%	60.01%	60.00%	59.98%	59.96%	59.96%	59.96%
14/2/07	60.16%	60.11%	60.07%	60.05%	60.04%	60.04%	60.03%	60.03%	60.02%	60.00%	59.99%	59.98%
14/2/08	60.12%	60.09%	60.07%	60.05%	60.05%	60.04%	60.04%	60.04%	60.03%	60.02%	60.01%	60.00%
14/2/10	60.09%	60.07%	60.06%	60.06%	60.05%	60.05%	60.05%	60.04%	60.04%	60.03%	60.03%	60.02%

BS with positive and negative jumps



Stochastic volatility

- Vol1 30%
- Vol2 40%
- Intensity 1 from 1 to 2
- Intensity 1 from 2 to 1

- Smile
- Long term flat

	Strike											
Maturity	5.00	7.50	10.00	12.50	14.00	15.00	16.00	17.50	20.00	25.00	30.00	40.00
1/3/05			44.68%	32.73%	30.48%	30.31%	30.41%	31.77%	37.88%			
14/3/05		50.99%	37.80%	31.66%	30.88%	30.77%	30.84%	31.31%	33.89%	40.52%		
14/5/05	46.77%	39.44%	34.98%	32.73%	32.34%	32.26%	32.30%	32.54%	33.38%	36.42%	39.25%	
14/8/05	41.64%	38.16%	35.51%	34.30%	34.04%	33.98%	34.00%	34.14%	34.61%	36.13%	37.87%	
14/2/06	40.42%	38.26%	36.99%	36.39%	36.25%	36.21%	36.21%	36.27%	36.48%	37.16%	37.97%	41.20%
14/2/07	40.09%	39.13%	38.63%	38.40%	38.33%	38.31%	38.30%	38.32%	38.39%	38.62%	38.90%	41.07%
14/2/08	40.18%	39.66%	39.40%	39.27%	39.23%	39.22%	39.22%	39.22%	39.25%	39.35%	39.49%	41.09%
14/2/10	40.41%	40.19%	40.08%	40.02%	40.01%	40.00%	39.99%	39.99%	40.00%	40.03%	40.08%	41.13%

Stochastic volatility

- Vol1 30%
- Vol2 40%
- Intensity .1 from 1 to 2
- Intensity .1 from 2 to 1

- Smile
- Long term

	Strike											
Maturity	5.00	7.50	10.00	12.50	14.00	15.00	16.00	17.50	20.00	25.00	30.00	40.00
1/3/05			43.65%	31.19%	30.05%	29.96%	30.00%	30.74%	34.56%			
14/3/05		50.87%	34.01%	30.23%	30.09%	30.06%	30.07%	30.17%	30.92%	37.09%		
14/5/05	46.27%	35.02%	30.94%	30.33%	30.27%	30.26%	30.26%	30.30%	30.46%	31.74%	34.70%	
14/8/05	37.64%	32.68%	30.94%	30.59%	30.53%	30.52%	30.52%	30.55%	30.66%	31.20%	32.40%	35.80%
14/2/06	35.11%	32.23%	31.35%	31.08%	31.03%	31.01%	31.01%	31.04%	31.12%	31.44%	31.98%	33.64%
14/2/07	34.29%	32.75%	32.19%	31.97%	31.91%	31.89%	31.89%	31.90%	31.96%	32.17%	32.48%	33.28%
14/2/08	34.49%	33.39%	32.93%	32.74%	32.69%	32.67%	32.66%	32.66%	32.70%	32.86%	33.08%	33.63%
14/2/10	35.25%	34.51%	34.18%	34.02%	33.97%	33.95%	33.93%	33.93%	33.94%	34.03%	34.17%	34.50%

Stochastic volatility and jumps

- Vol1 30%, Vol2 40%
- Intensities 1
- Jump 15% from 1 to 2
- Jump –15% from 2 to 1

- Positive skew
- Long term

Maturity	Strike											
	5.00	7.50	10.00	12.50	14.00	15.00	16.00	17.50	20.00	25.00	30.00	40.00
1/3/05			43.57%	31.83%	31.39%	32.41%	35.16%	43.29%	51.44%			
14/3/05		50.97%	33.18%	31.51%	32.23%	33.18%	34.78%	38.57%	44.81%	50.07%		
14/5/05	46.51%	35.22%	32.98%	33.49%	34.30%	35.01%	35.87%	37.39%	40.14%	44.37%	46.62%	
14/8/05	38.06%	35.23%	34.87%	35.64%	36.37%	36.93%	37.53%	38.49%	40.11%	42.88%	44.75%	46.82%
14/2/06	37.69%	37.12%	37.56%	38.38%	38.95%	39.34%	39.73%	40.32%	41.25%	42.84%	44.06%	45.66%
14/2/07	39.26%	39.70%	40.32%	40.95%	41.32%	41.55%	41.77%	42.10%	42.59%	43.42%	44.08%	45.07%
14/2/08	40.49%	41.05%	41.58%	42.06%	42.31%	42.47%	42.63%	42.84%	43.17%	43.72%	44.16%	44.84%
14/2/10	41.86%	42.32%	42.68%	42.98%	43.14%	43.23%	43.33%	43.45%	43.64%	43.97%	44.23%	44.64%

Stochastic volatility and jumps

- Vol1 30%, Vol2 40%
- Intensities 1
- Jump -15% from 1 to 2
- Jump 15% from 2 to 1
- Negative skew
- Long term

Maturity	Strike											
	5.00	7.50	10.00	12.50	14.00	15.00	16.00	17.50	20.00	25.00	30.00	40.00
1/3/05			56.96%	47.77%	36.16%	32.58%	31.45%	31.54%	33.84%			
14/3/05		53.96%	49.62%	41.23%	35.44%	33.41%	32.36%	31.64%	31.40%	34.20%		
14/5/05	50.28%	47.27%	43.25%	38.40%	36.20%	35.16%	34.39%	33.61%	32.87%	32.68%	33.96%	
14/8/05	47.74%	45.06%	41.88%	38.95%	37.60%	36.88%	36.30%	35.61%	34.84%	34.19%	34.29%	35.78%
14/2/06	46.18%	44.00%	41.98%	40.30%	39.49%	39.02%	38.61%	38.09%	37.42%	36.61%	36.24%	36.23%
14/2/07	45.09%	43.68%	42.57%	41.68%	41.23%	40.96%	40.72%	40.40%	39.94%	39.26%	38.80%	38.28%
14/2/08	44.60%	43.60%	42.85%	42.25%	41.95%	41.77%	41.61%	41.38%	41.05%	40.53%	40.14%	39.62%
14/2/10	44.14%	43.52%	43.07%	42.72%	42.54%	42.43%	42.33%	42.20%	41.99%	41.66%	41.40%	41.00%

Black-Scholes and default

- Brownian vol 40%
- Default intensity .1
- Recovery 0 or 50%

- Flat spreads
- Depend on recovery

Maturity	Spread R=0	Spread R=.5
1	10.07%	5.04%
2	10.07%	5.04%
3	10.07%	5.04%
5	10.07%	5.04%
10	10.07%	5.04%

Black-Scholes and default

- Brownian vol 40%
- Default intensity .1
- Jump to 0 on default

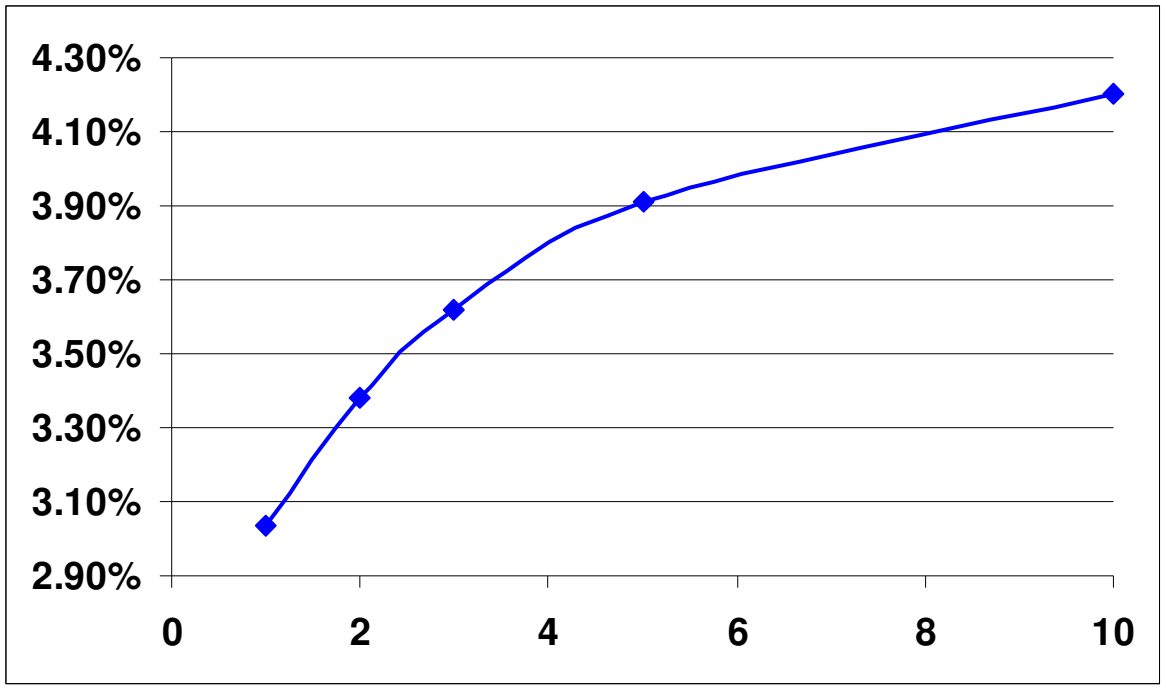
- Large short term skew
- Long term skew
- Higer total vol

Maturity	Strike											
	5.00	7.50	10.00	12.50	14.00	15.00	16.00	17.50	20.00	25.00	30.00	40.00
1/3/05			117.5%	64.14%	45.42%	42.27%	41.28%	41.13%				
14/3/05		139.3%	93.1%	55.92%	45.86%	43.43%	42.30%	41.54%	41.12%			
14/5/05	136.9%	99.9%	71.1%	53.00%	48.02%	46.12%	44.88%	43.71%	42.64%	41.75%		
14/8/05	113.1%	85.4%	65.5%	54.04%	50.33%	48.65%	47.40%	46.06%	44.67%	43.27%	42.59%	
14/2/06	96.99%	77.10%	64.12%	56.45%	53.54%	52.06%	50.86%	49.45%	47.80%	45.88%	44.82%	43.71%
14/2/07	87.34%	73.97%	65.50%	60.08%	57.77%	56.51%	55.42%	54.05%	52.29%	49.98%	48.54%	46.86%
14/2/08	84.21%	73.84%	67.20%	62.75%	60.75%	59.62%	58.62%	57.34%	55.61%	53.20%	51.60%	49.61%
14/2/10	82.38%	74.93%	70.05%	66.59%	64.97%	64.02%	63.17%	62.04%	60.46%	58.11%	56.43%	54.19%

Stochastic default

- Brownian vol 40%
- Def1=.05 and Def2=.15
- Int .25 from 1 to 2
- Int .25 from 2 to 1

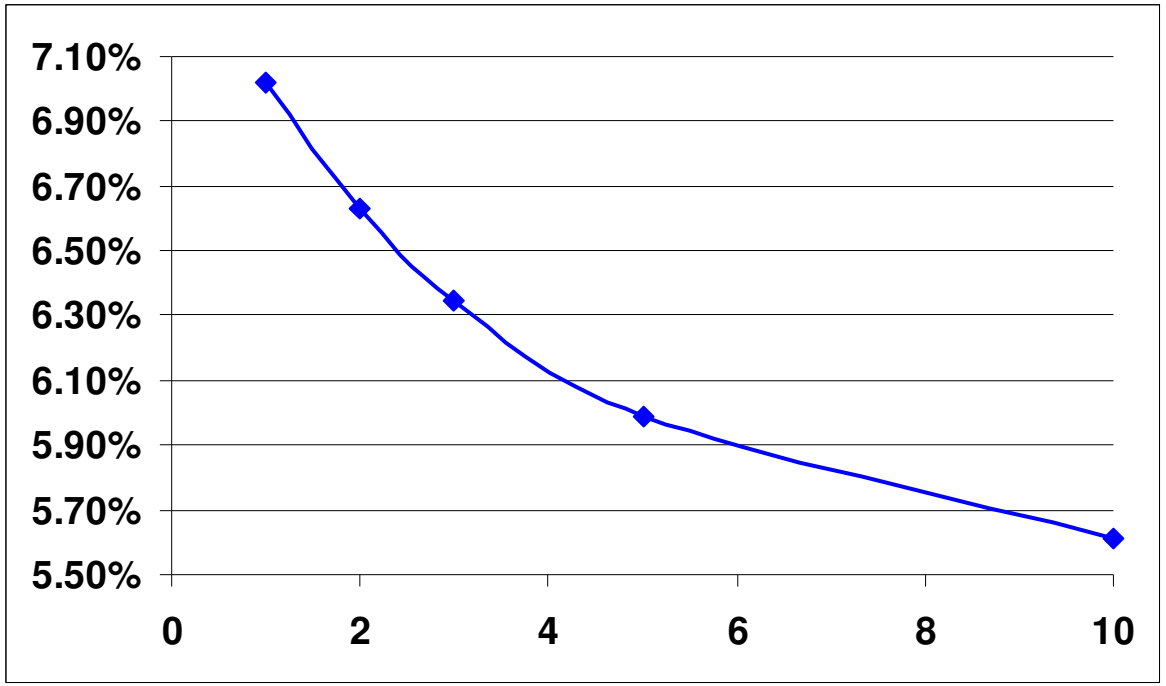
Maturity	Spread
1	3.03%
2	3.38%
3	3.62%
5	3.91%
10	4.20%



Stochastic default

- Brownian vol 40%
- Def1=.15 and Def2=.05
- Int .25 from 1 to 2
- Int .25 from 2 to 1

Maturity	Spread
1	7.02%
2	6.63%
3	6.35%
5	5.99%
10	5.61%

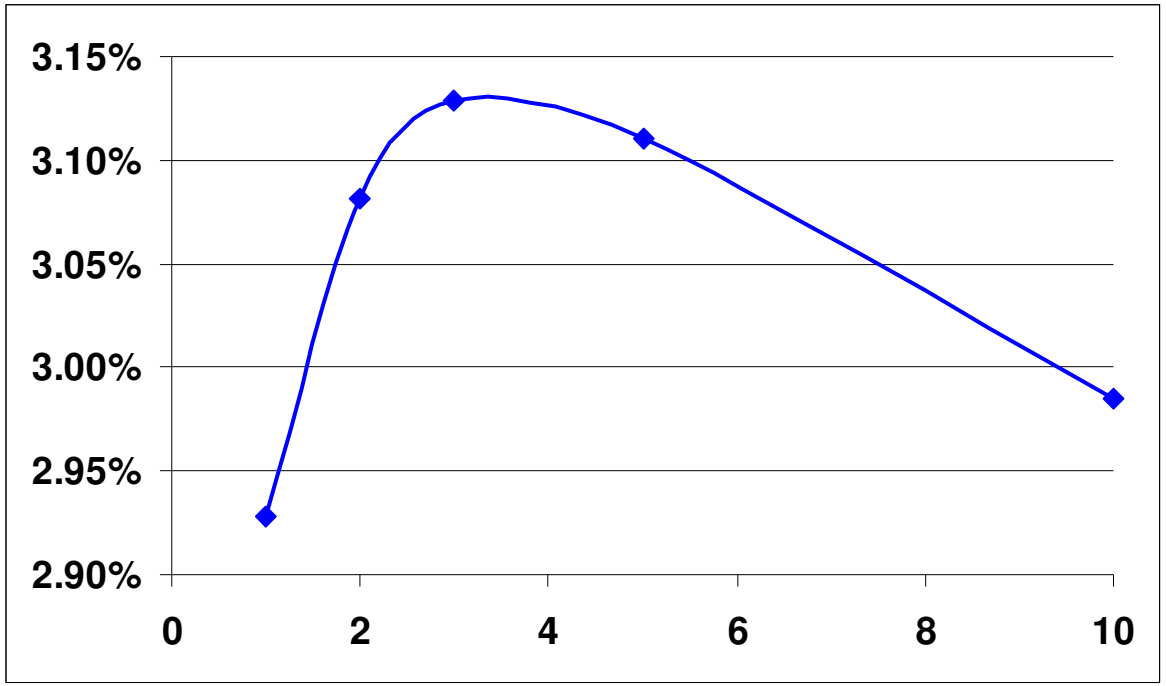


Stochastic default

- Default Int1 = .05
- Default Int2= .1
- Default Int3 = .05

- Int .25 from 1 to 2
- Int 1 from 2 to 3

Maturity	Spread
1	2.93%
2	3.08%
3	3.13%
5	3.11%
10	2.98%



Hedging Default

Hedging Default

- Infinitesimal spread variations
  - day to day P/L management
- Jumps in spreads
  - downgrades or corporate news
- Jump to default
  - Infrequent but catastrophic
  - May not be predictable (Enron, Parmalat)

Hedging Default

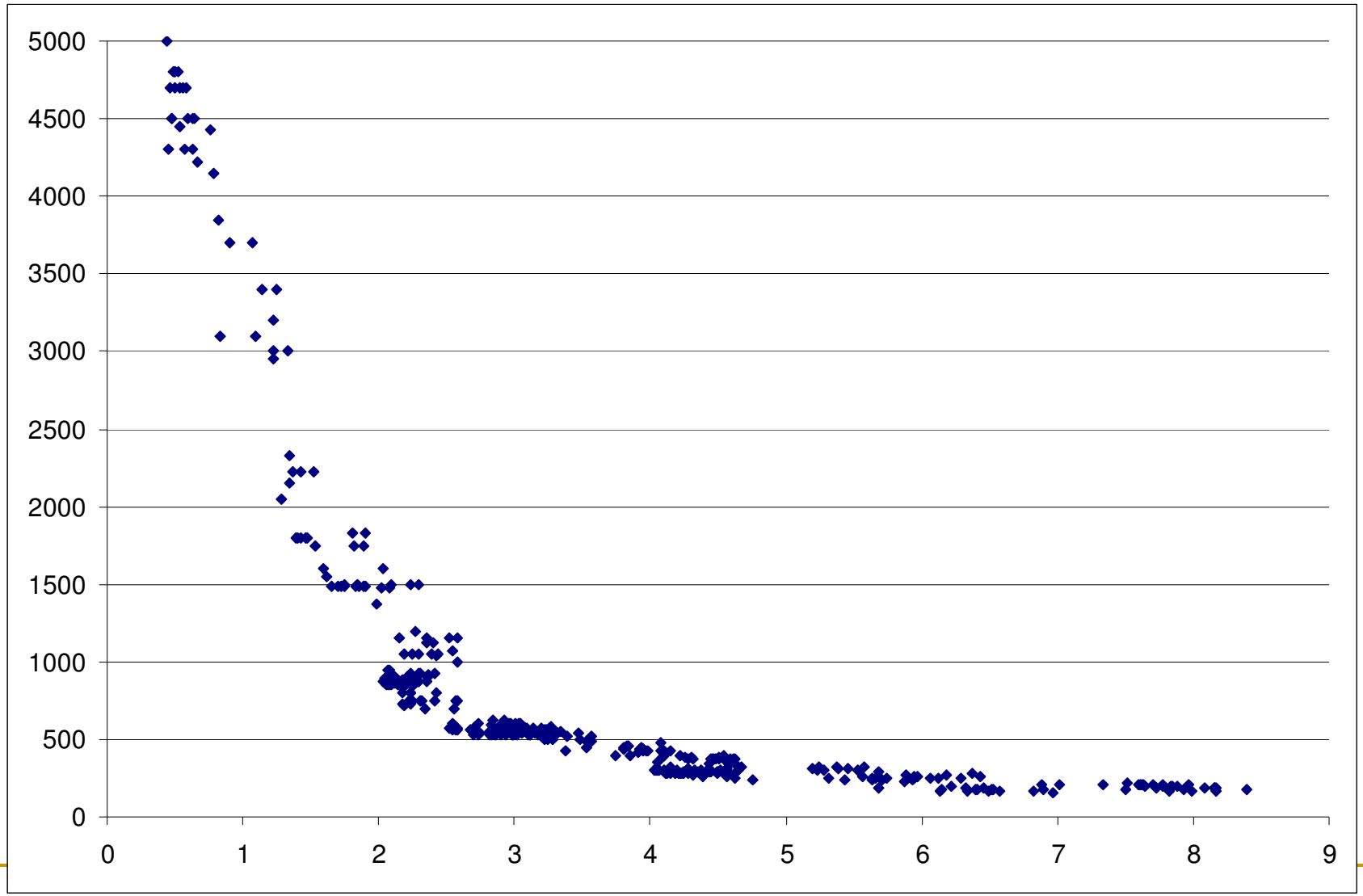
Model	Black Scholes	ihg	hg
	Diffusion	Diffusion & default jump	Regimes
Hedging	Delta	Delta & CDS	Multi instruments
Infinitesimal	✓	✓	✓
Spread jumps			✓
Jump to Default		✓	✓

Spread – Spot

CDS Spreads a function of Spot?

Sometime OK

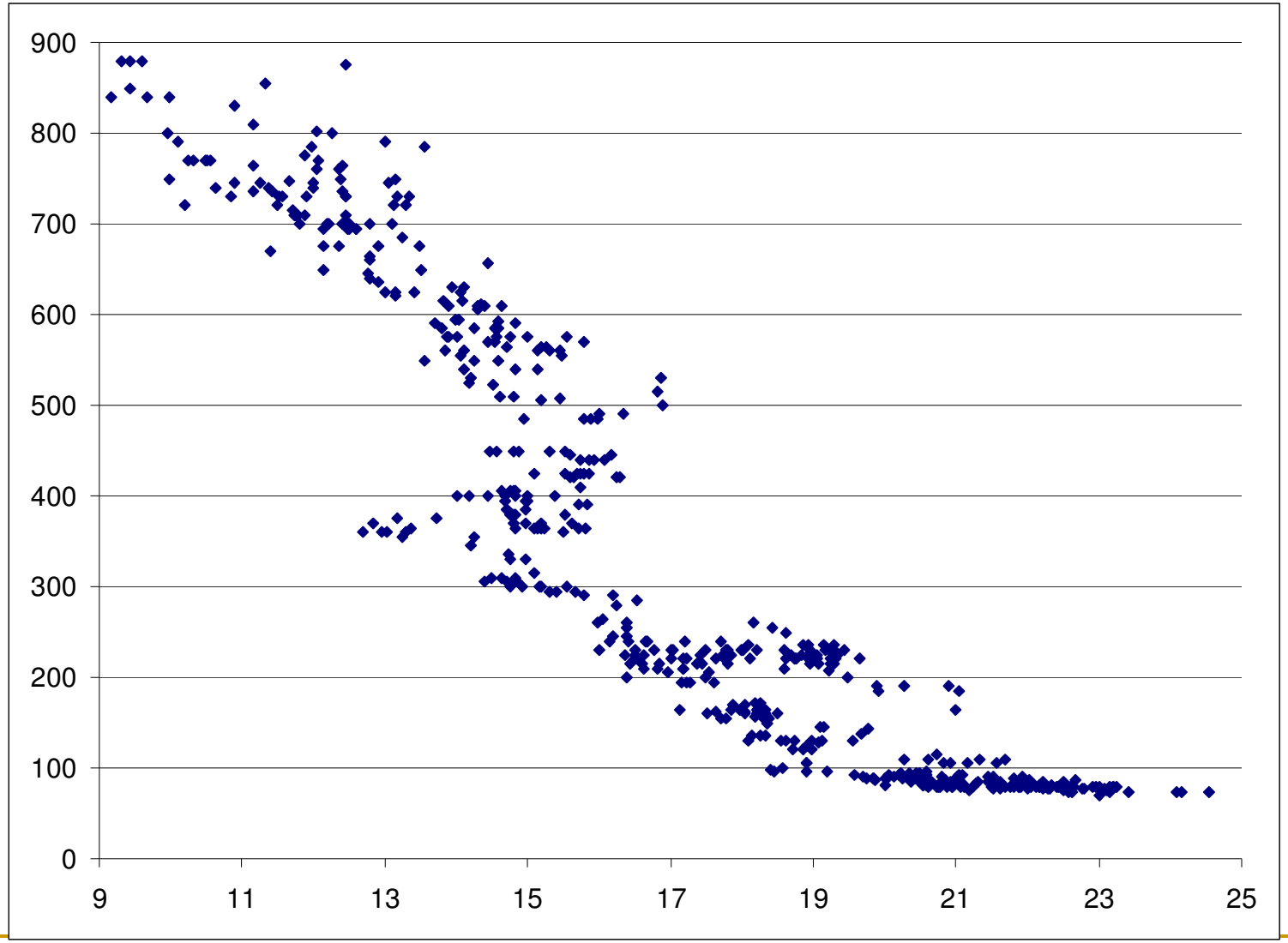
NT


5000
4500
4000
3500
3000
2500
2000
1500
1000
500
0
0
1
2
3
4
5
6
7
8
9

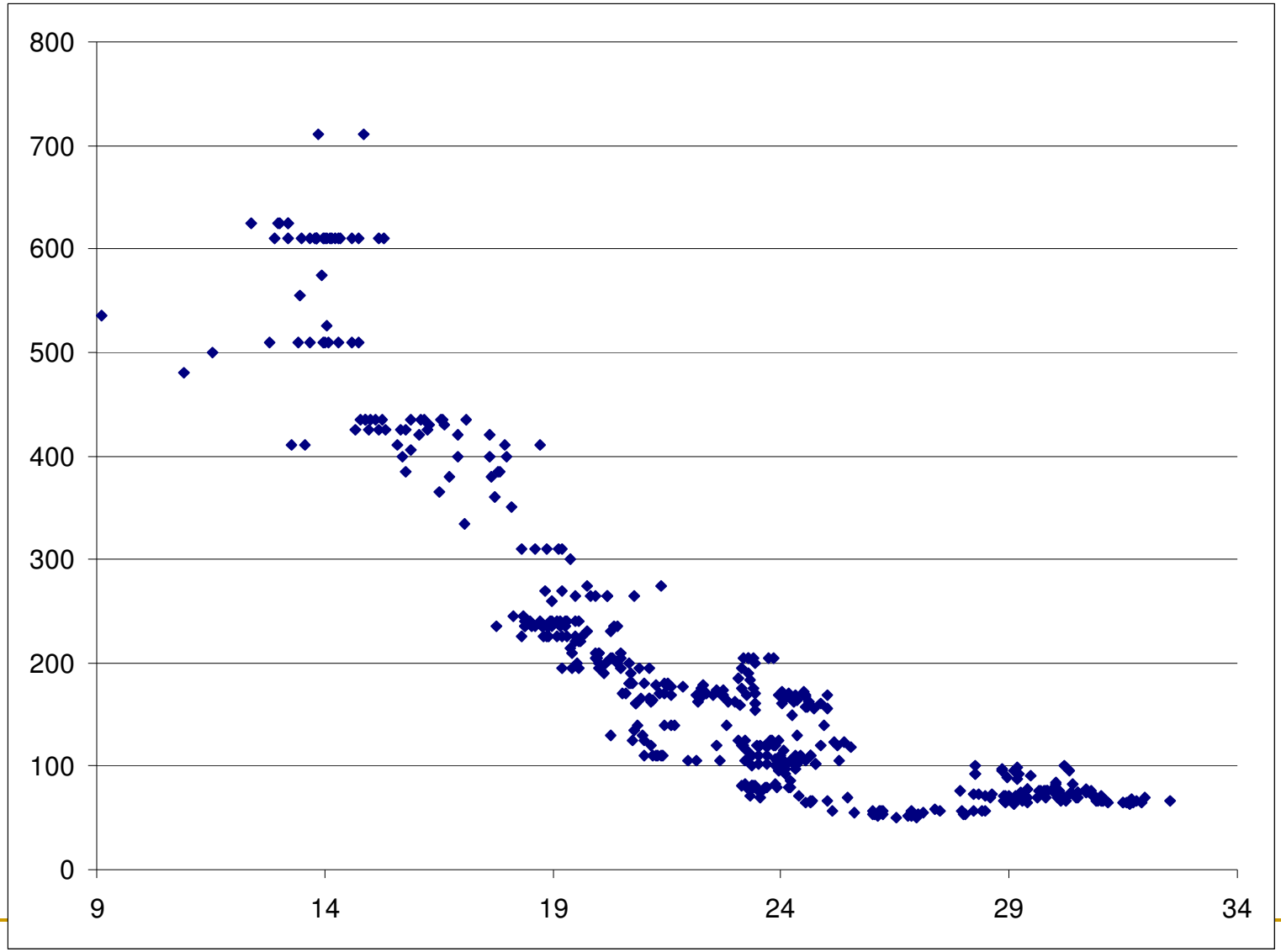
Spread – Spot

Sometime challenging

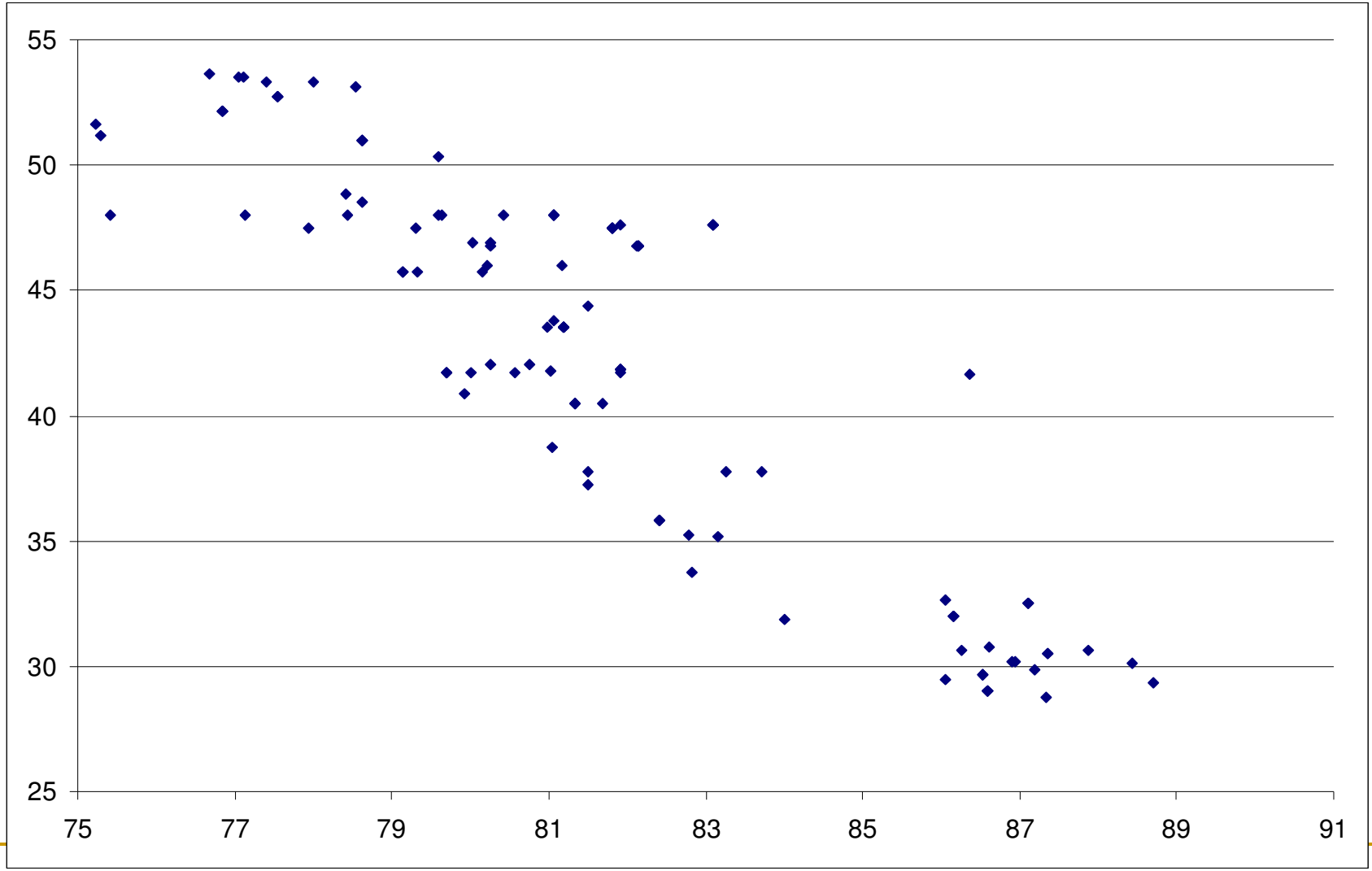
GPS



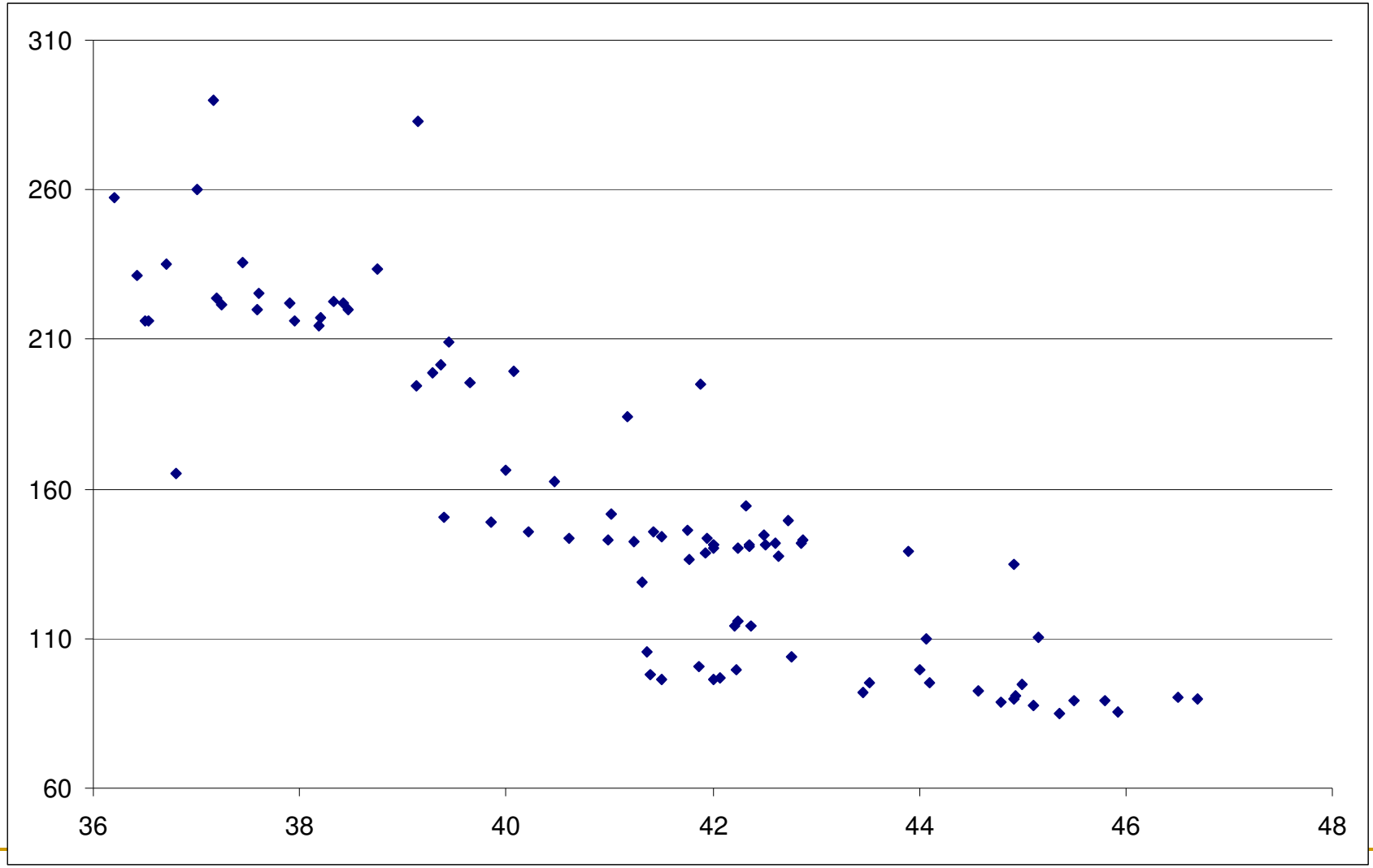
HAL



OMN



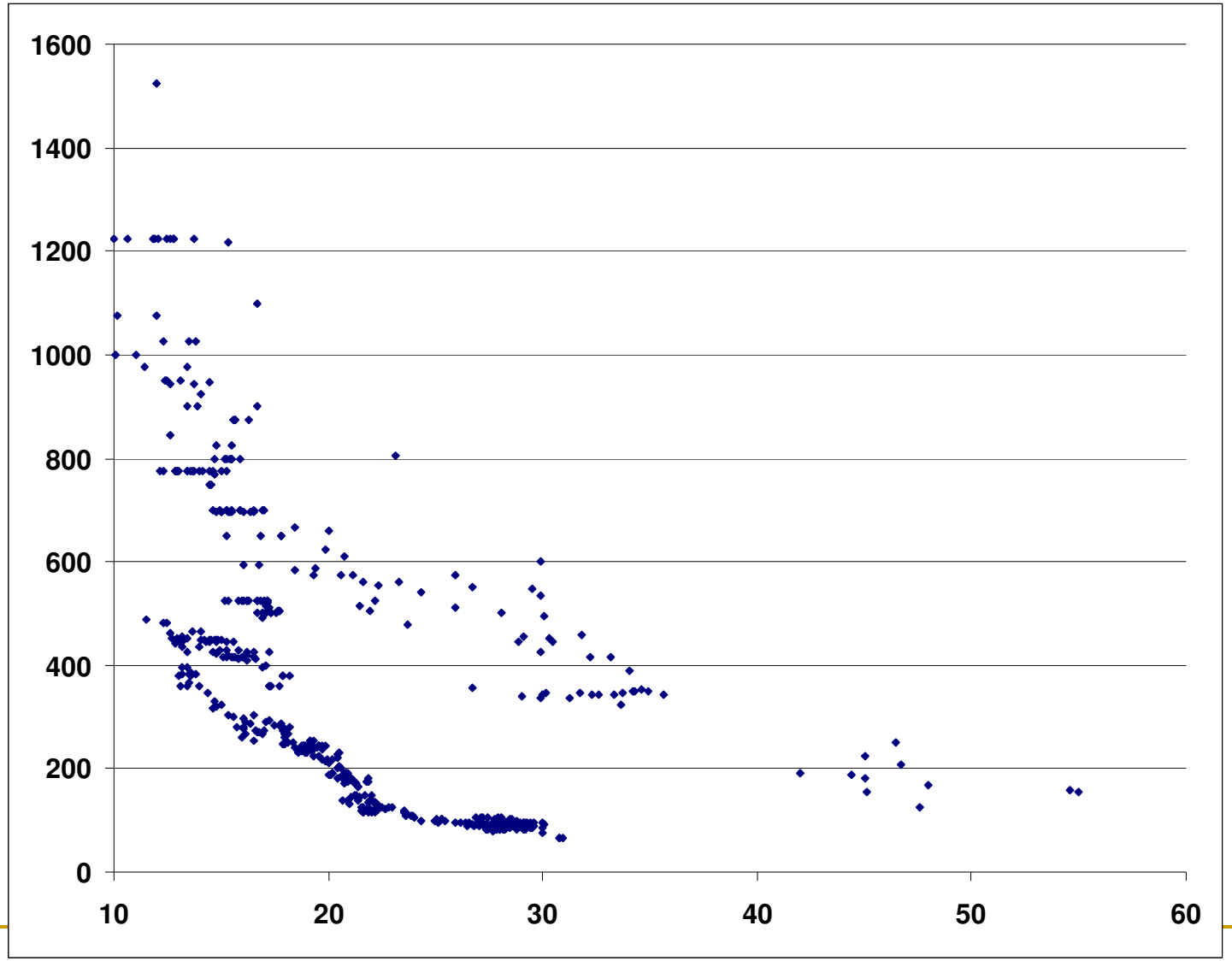
ADE



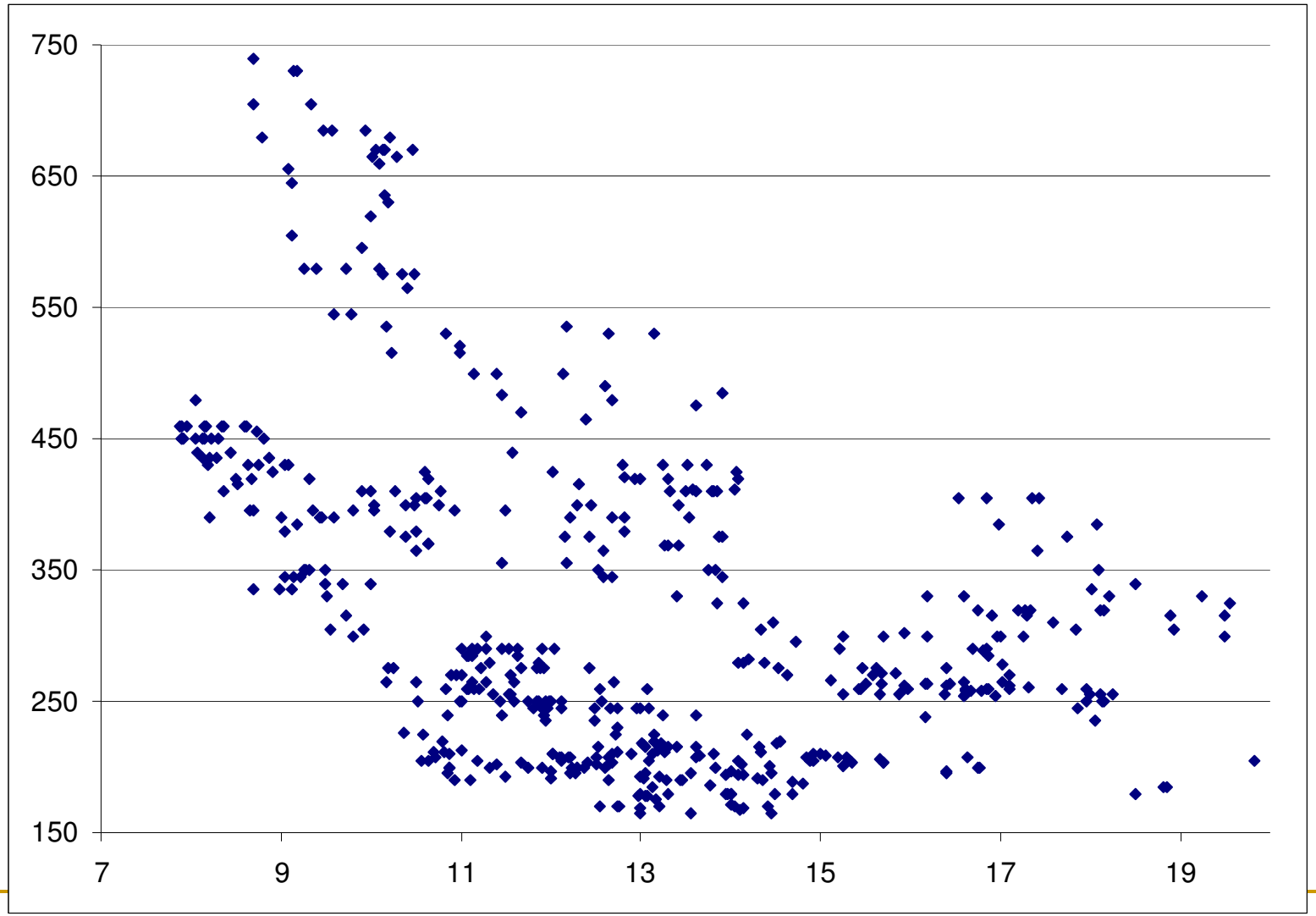
Spread – Spot

More challenging

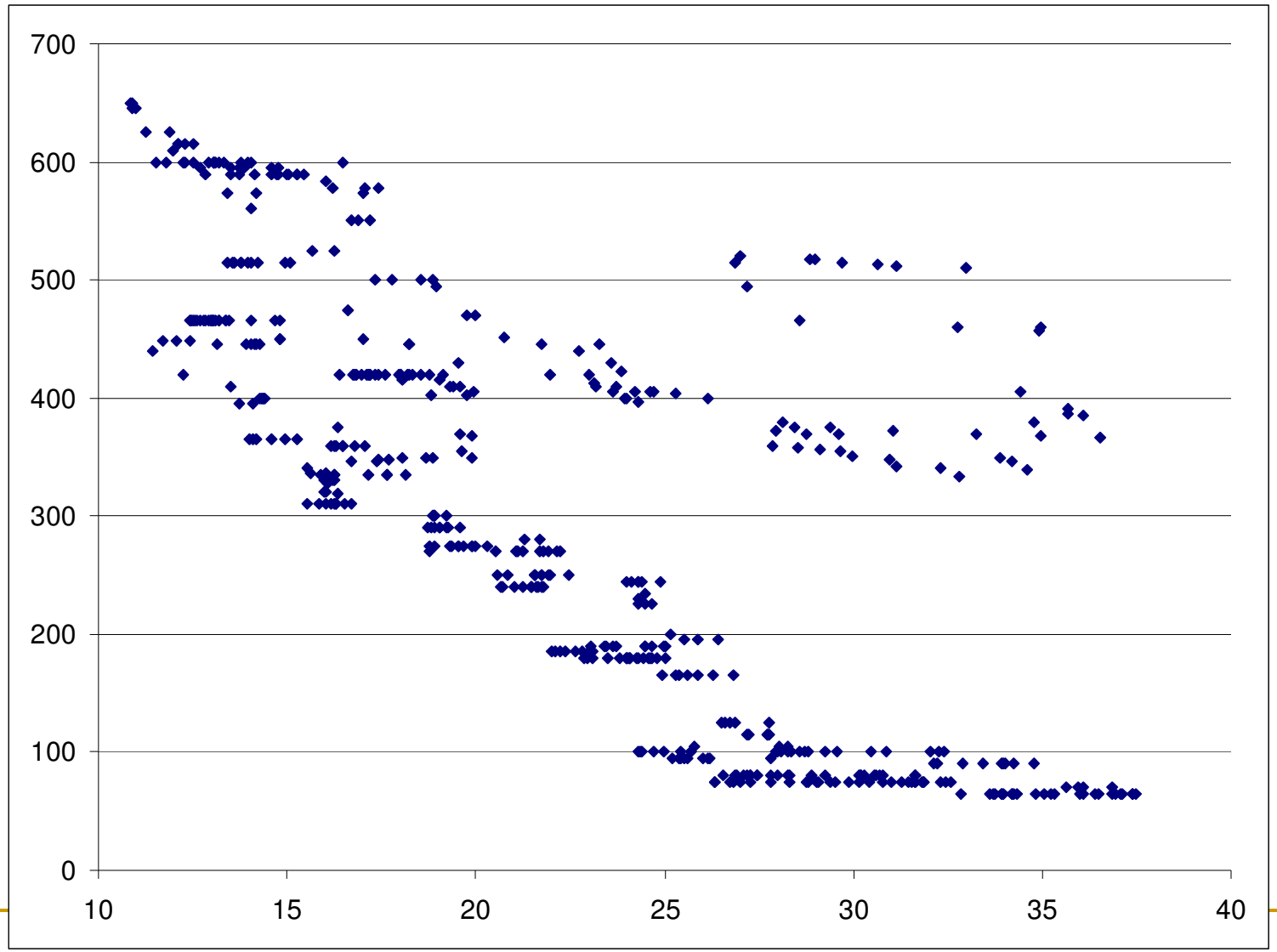
TYC



TOY



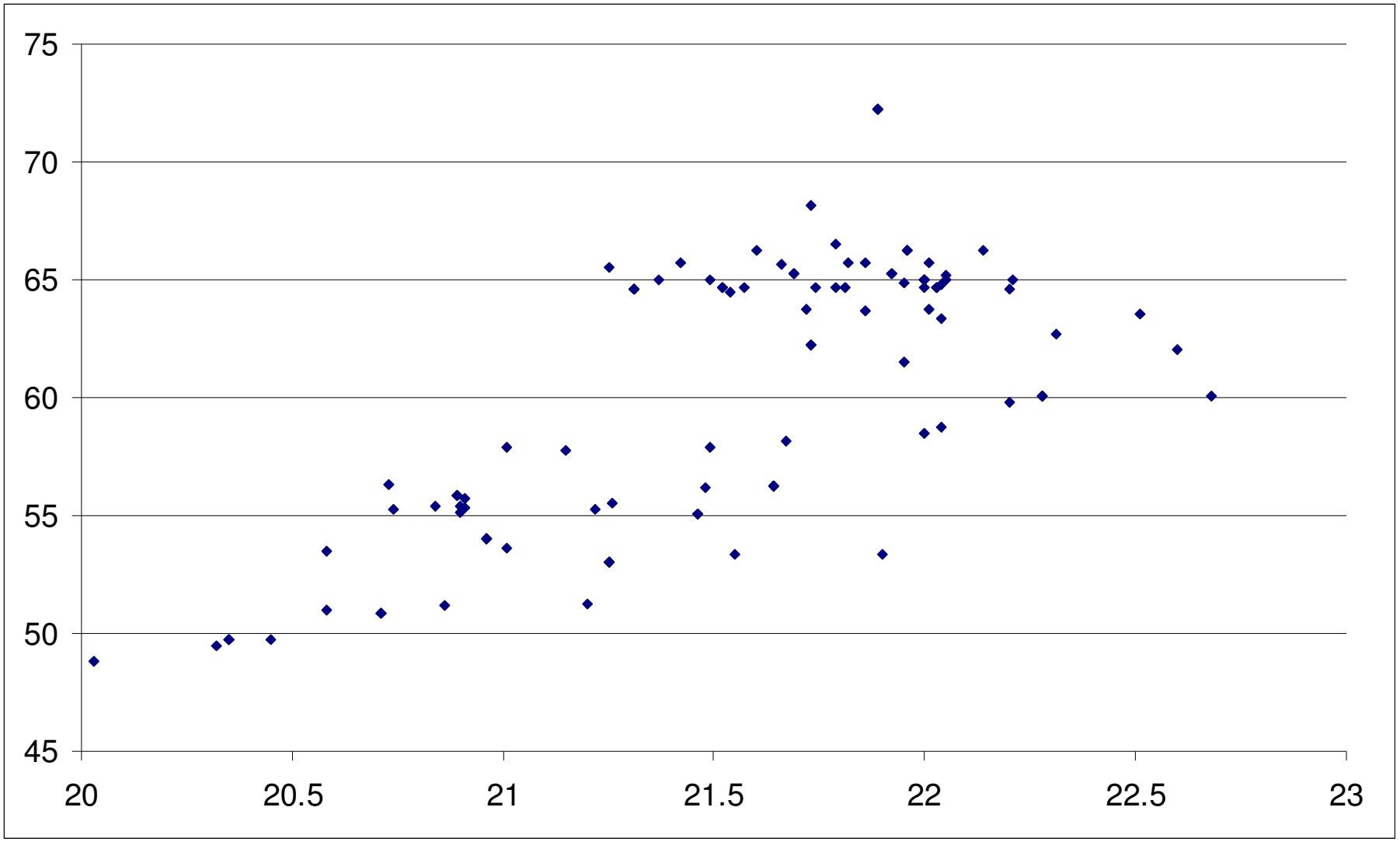
A.N



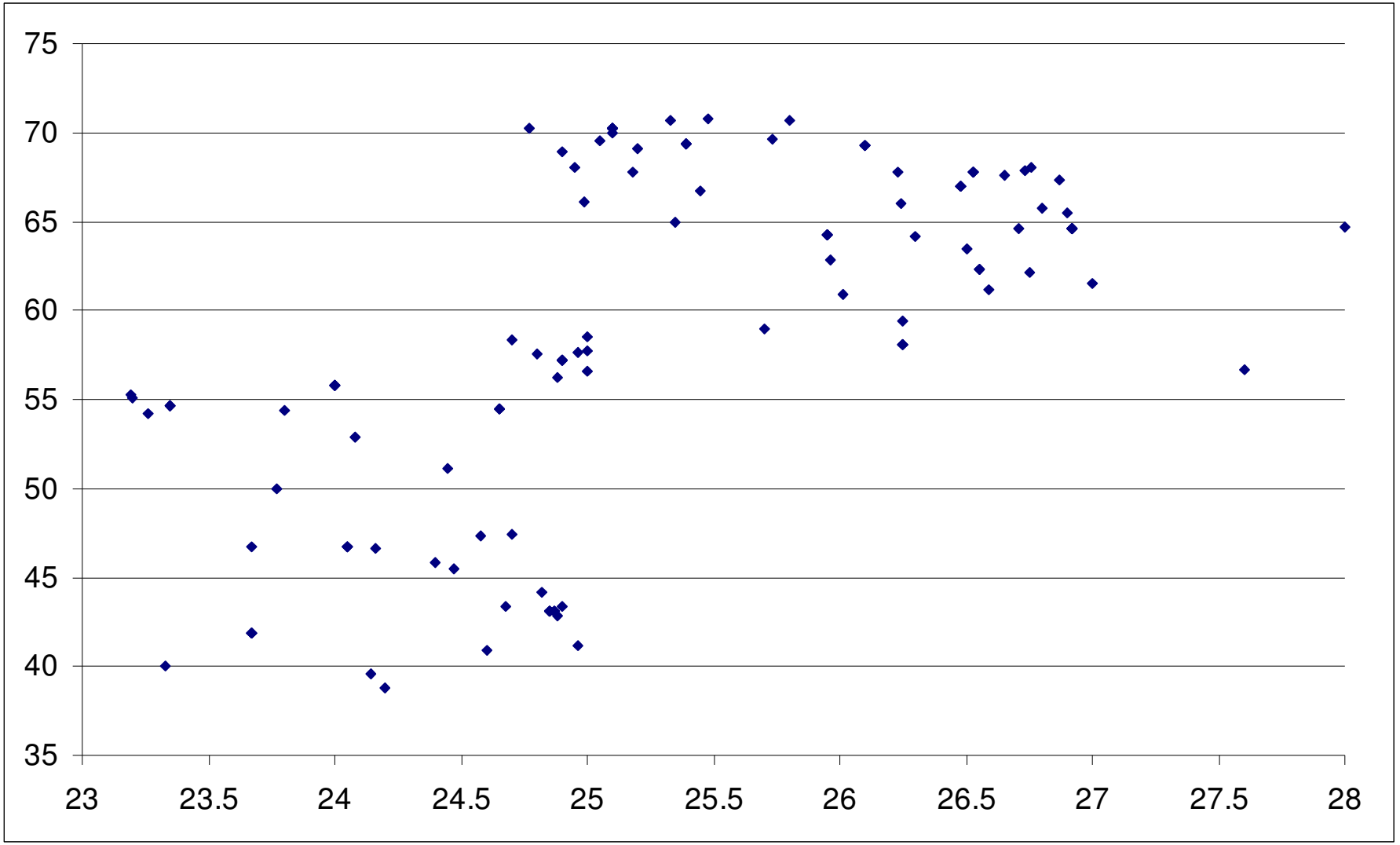
Spread – Spot

Wrong way!

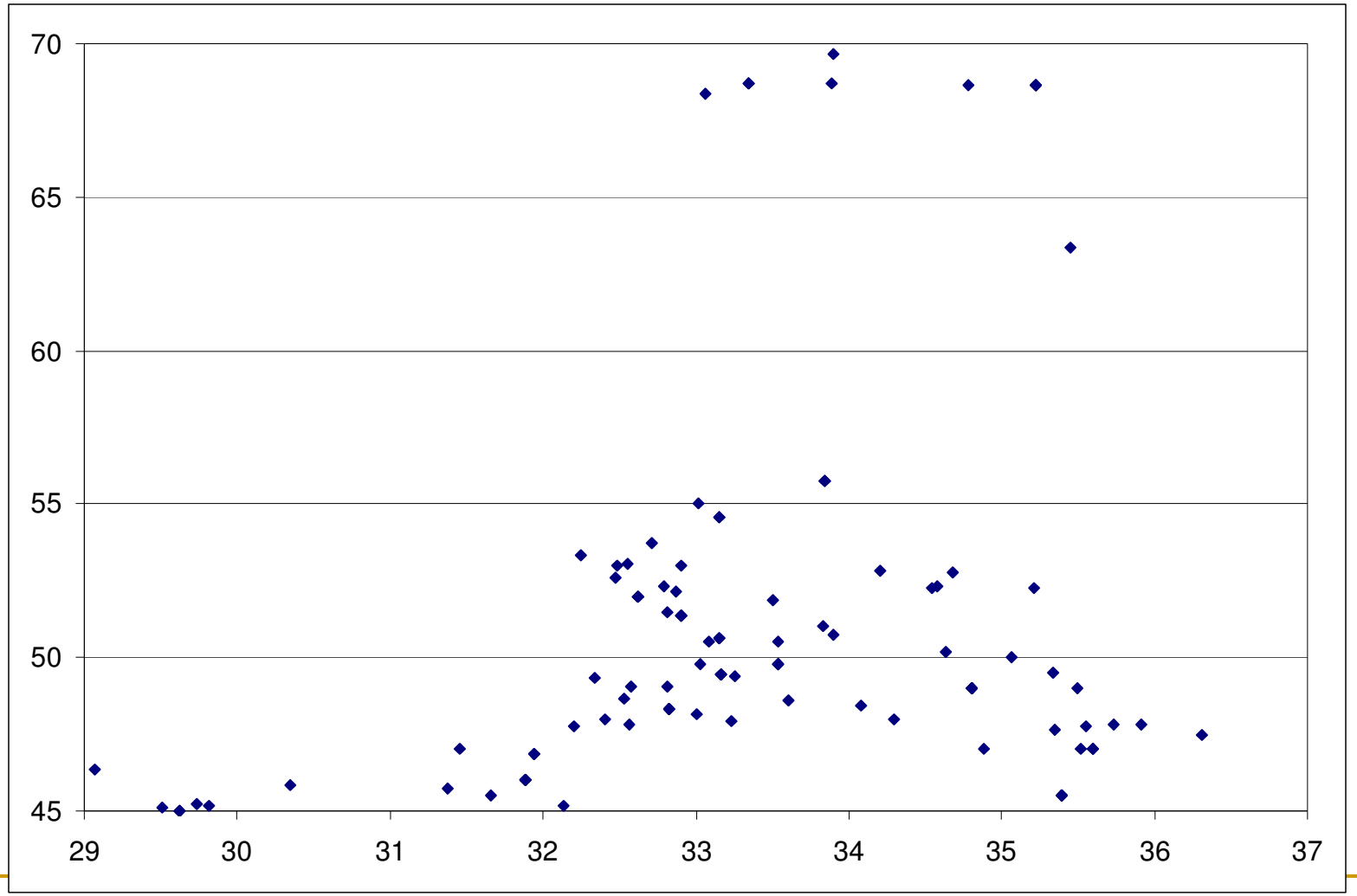
DUK



DIS



MAY



Hedging Volatility

Hedging Volatility

- Traditional infinitesimal Delta hedging of Gamma positions
  - day to day P&L management
- Infinitesimal movement in volatility unrelated to the underlying
  - stochastic volatility
- Jumps in volatility
  - large corporate events

Hedging Volatility

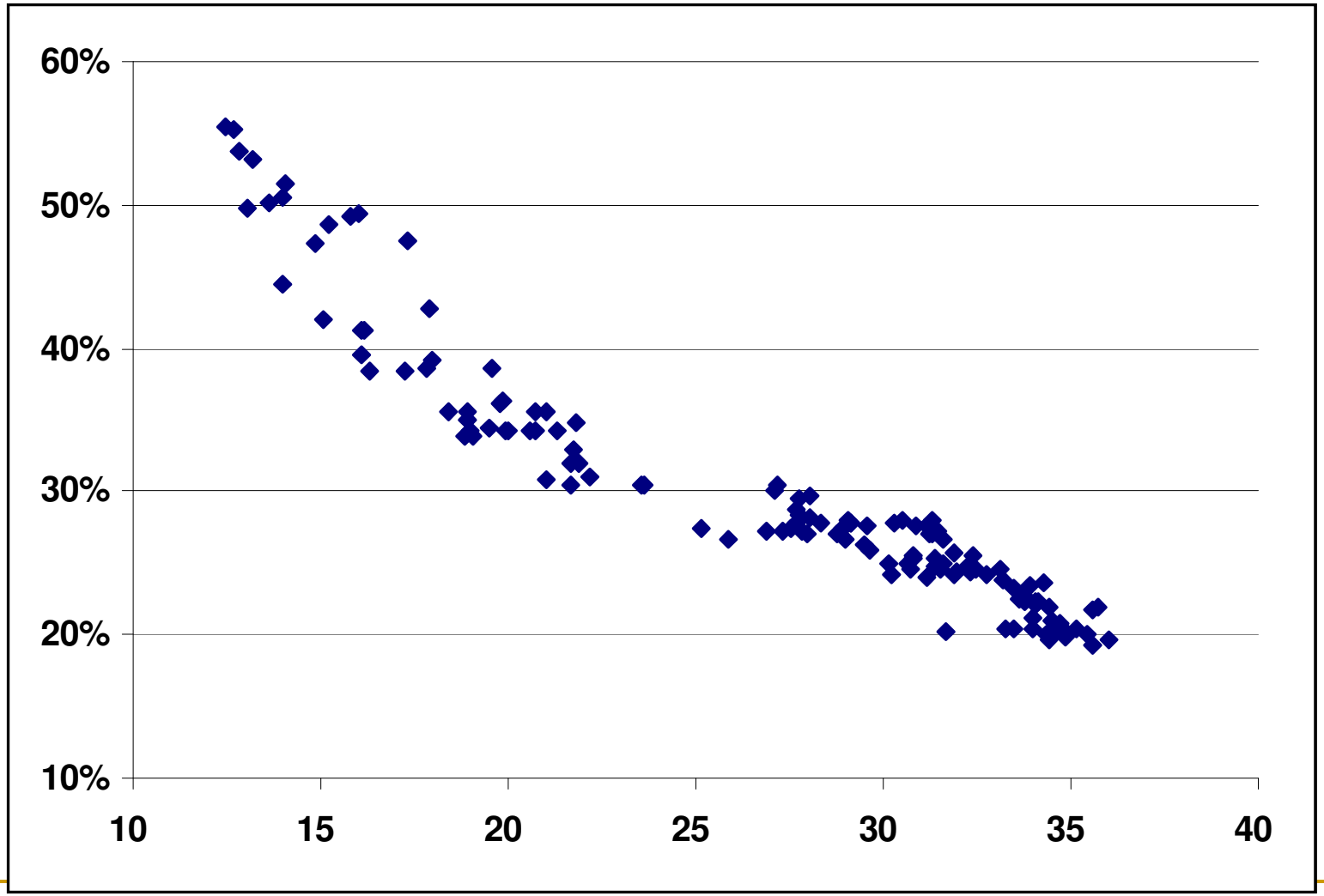
	Black Scholes	ihg	hg
Model	**Diffusion**	**Diffusion & default jump**	**Regimes**
Hedging	**Delta**	**Delta**	**Multi instruments**
Gamma	✓	✓	✓
Stochastic vol			✓
Jumps in vol			✓

Vol – Spot

ATM Implied vol a function of Spot?

Sometime OK

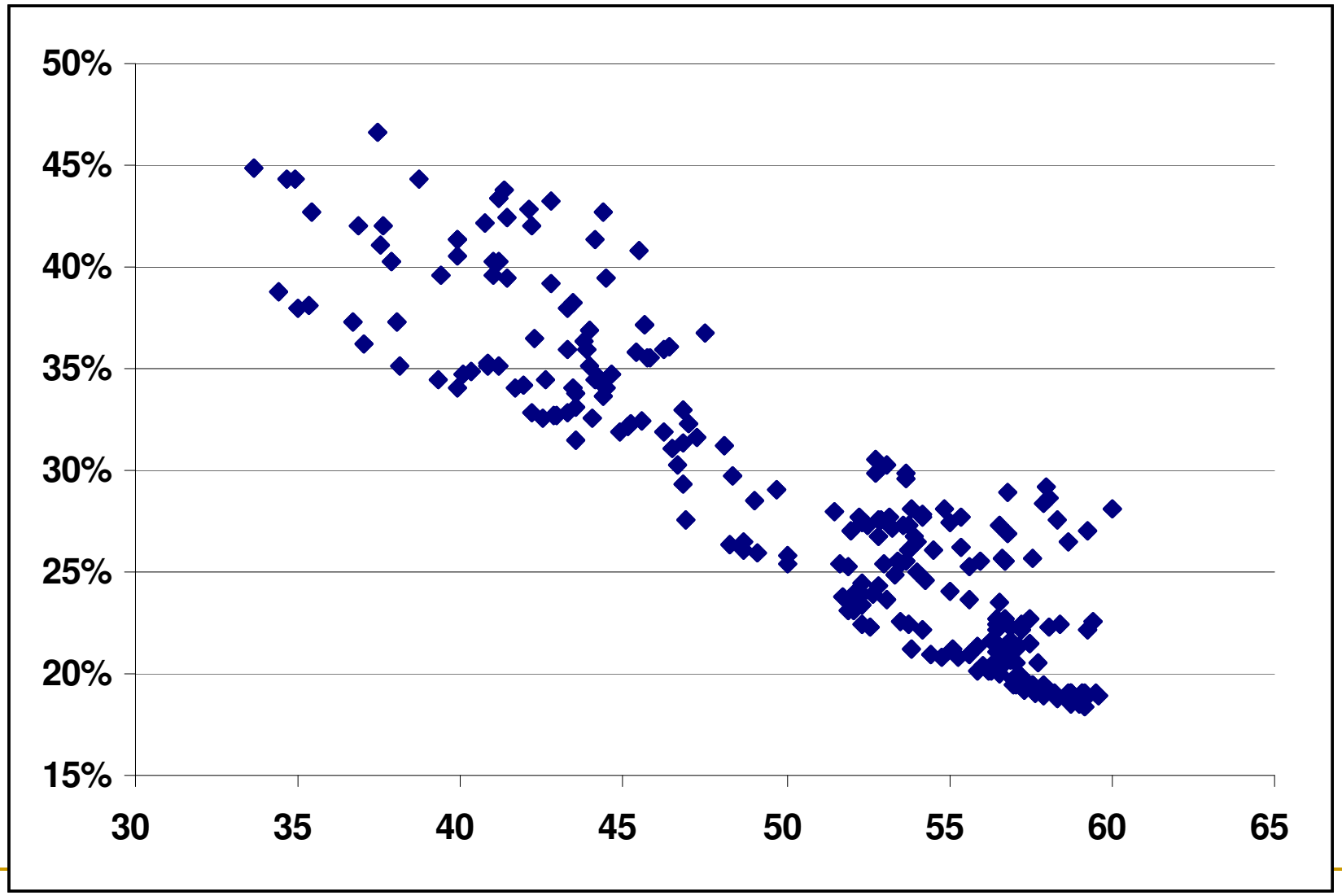
TYC



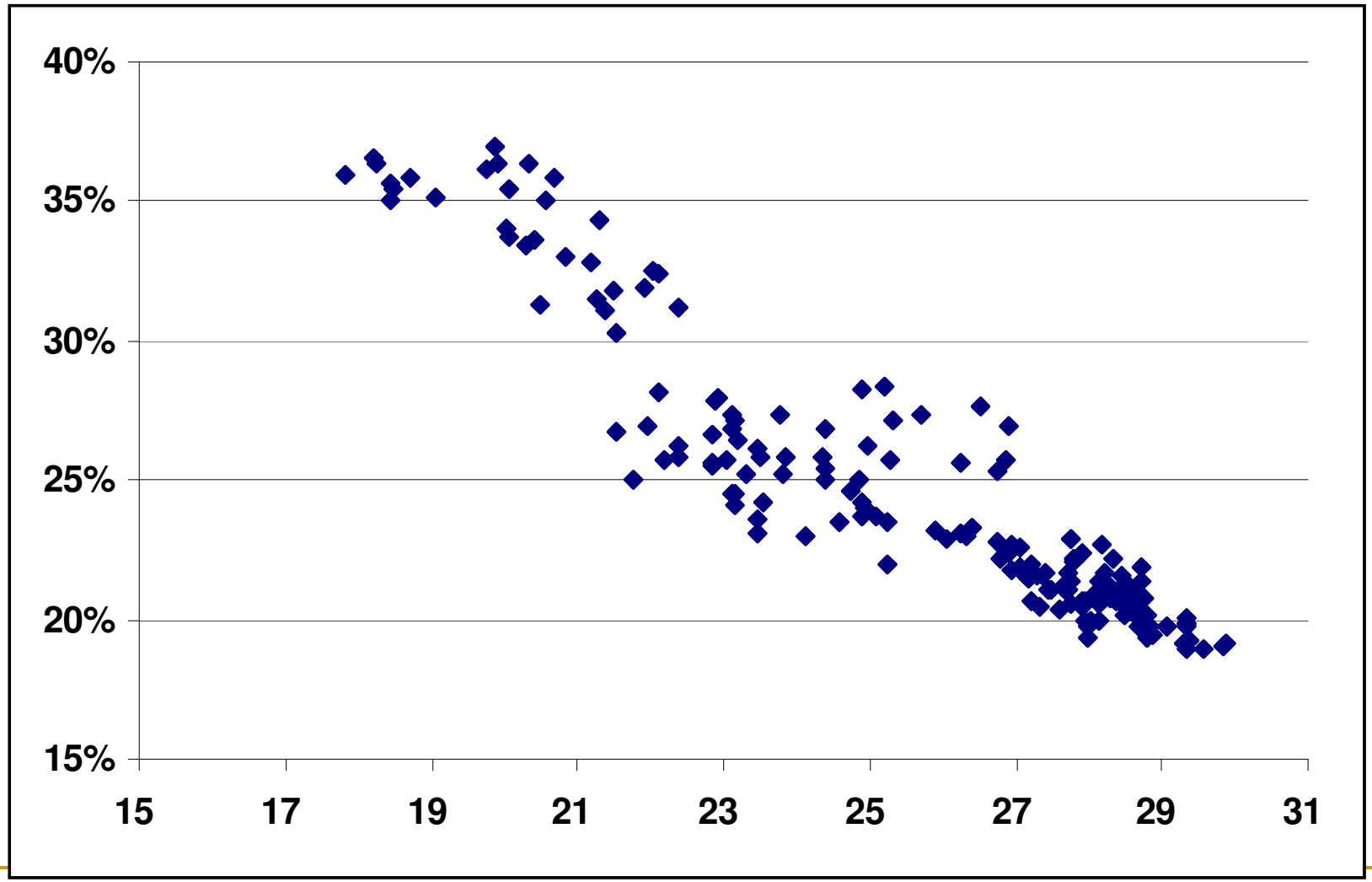
Vol – Spot

Sometime less OK

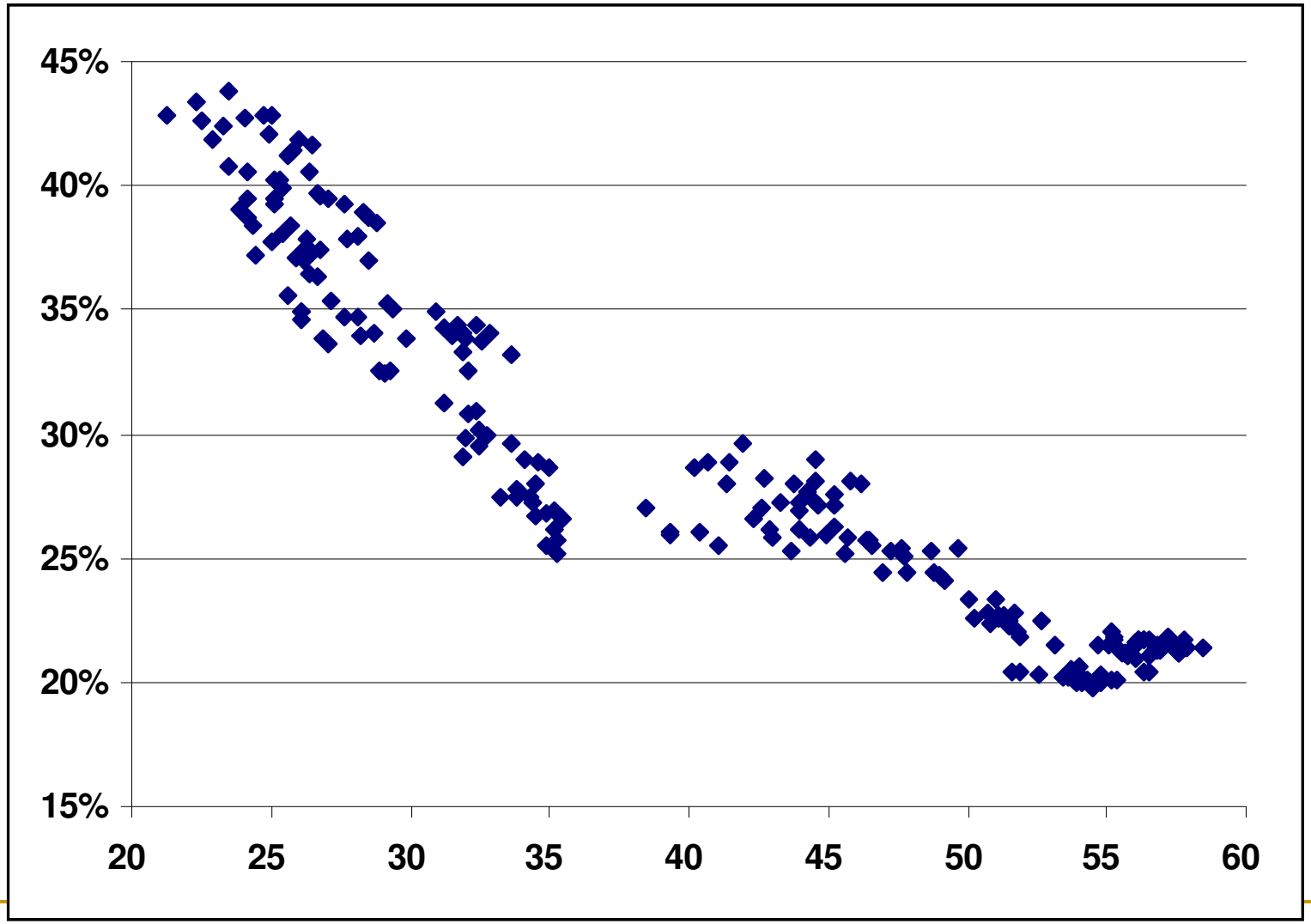
LOW



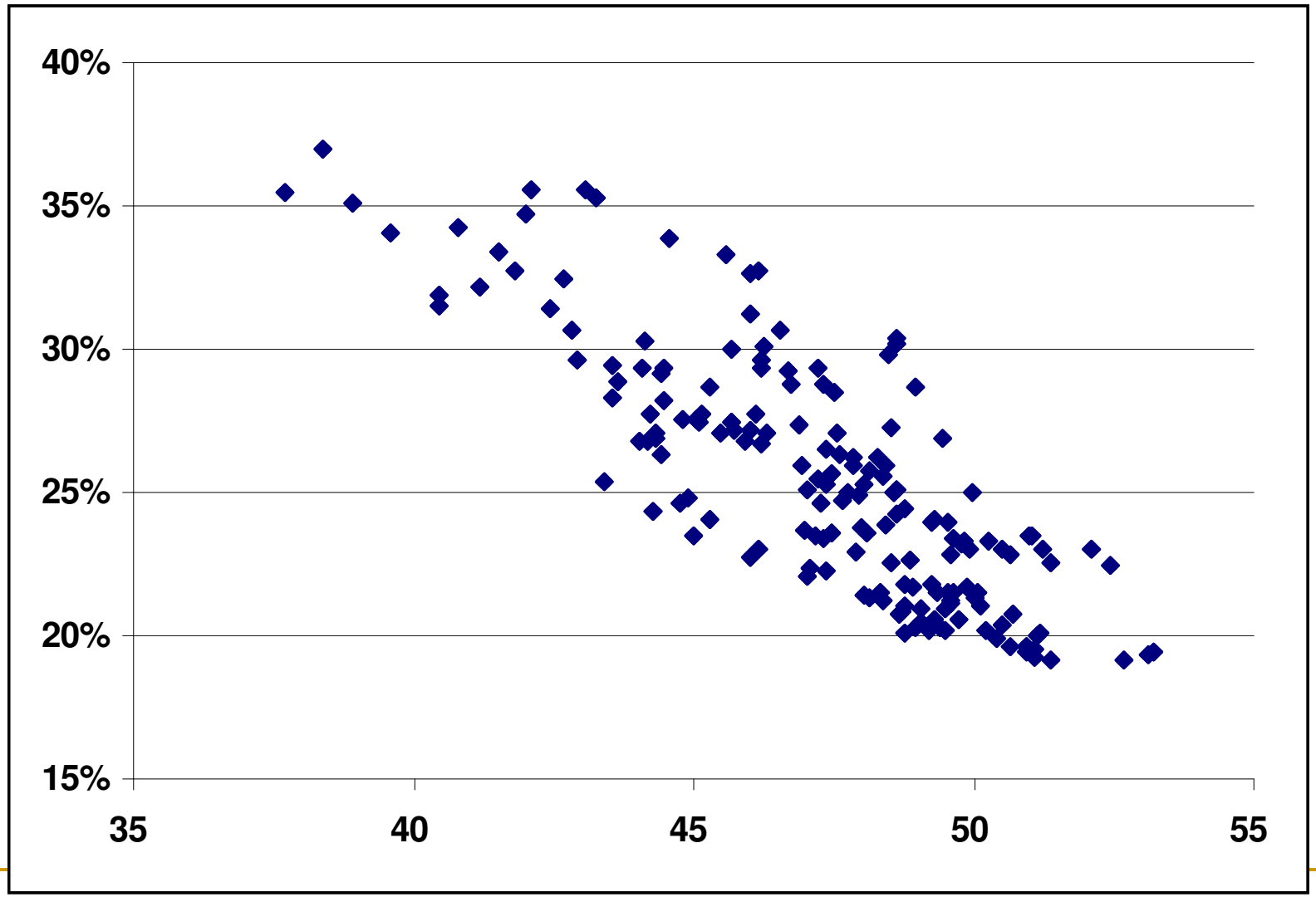
DIS



CCL



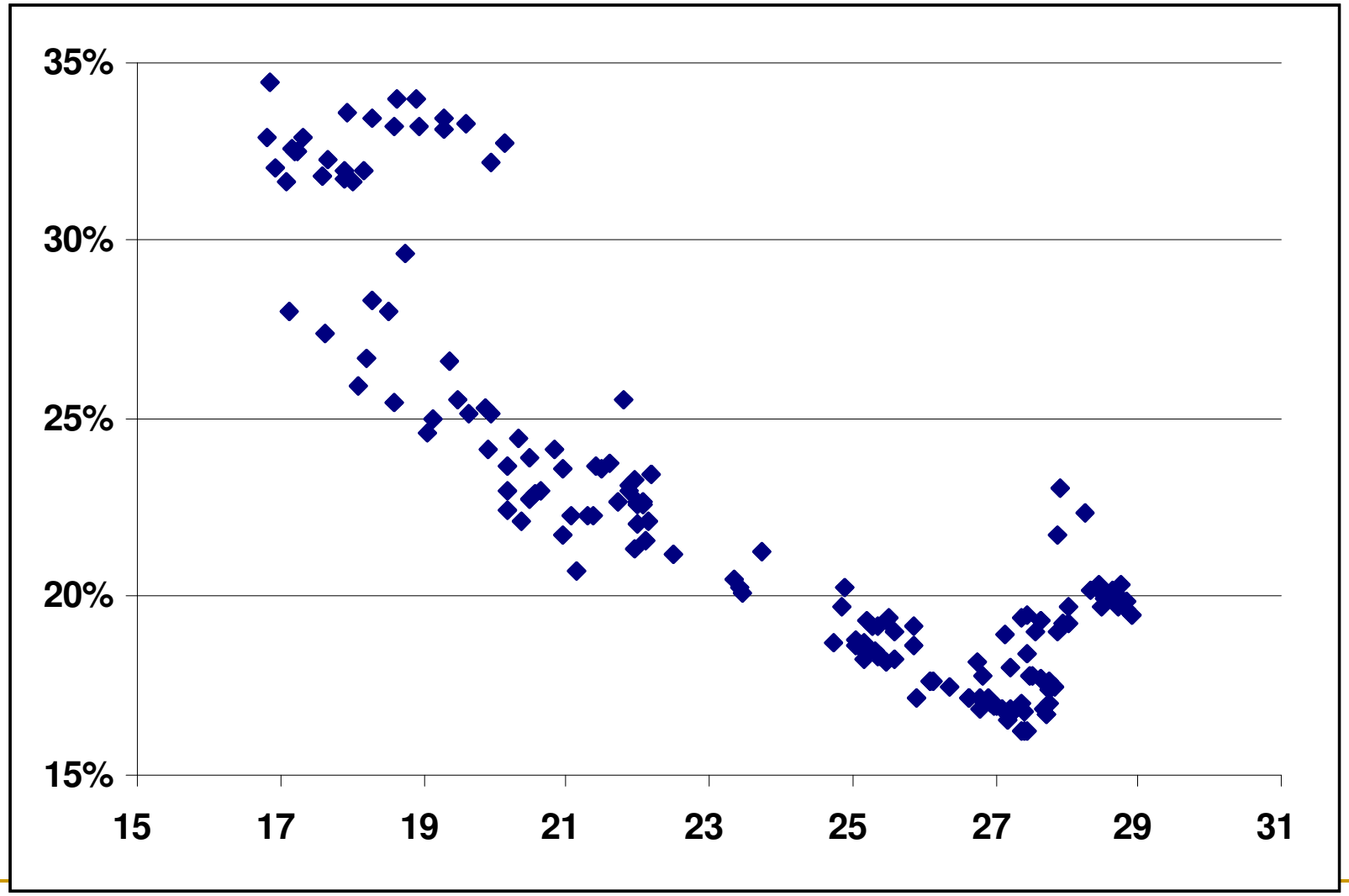
MDT



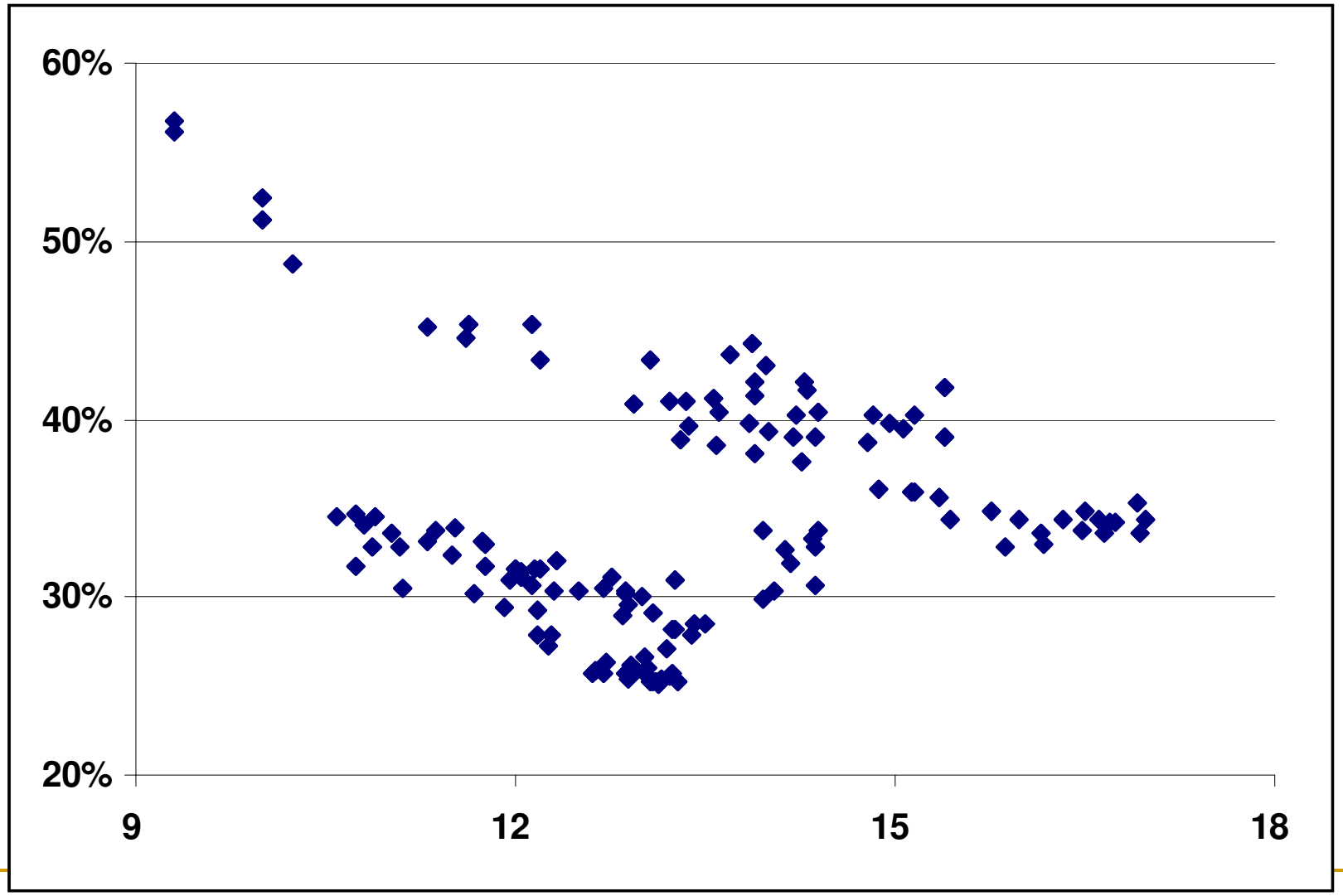
Vol – Spot

Sometime challenging

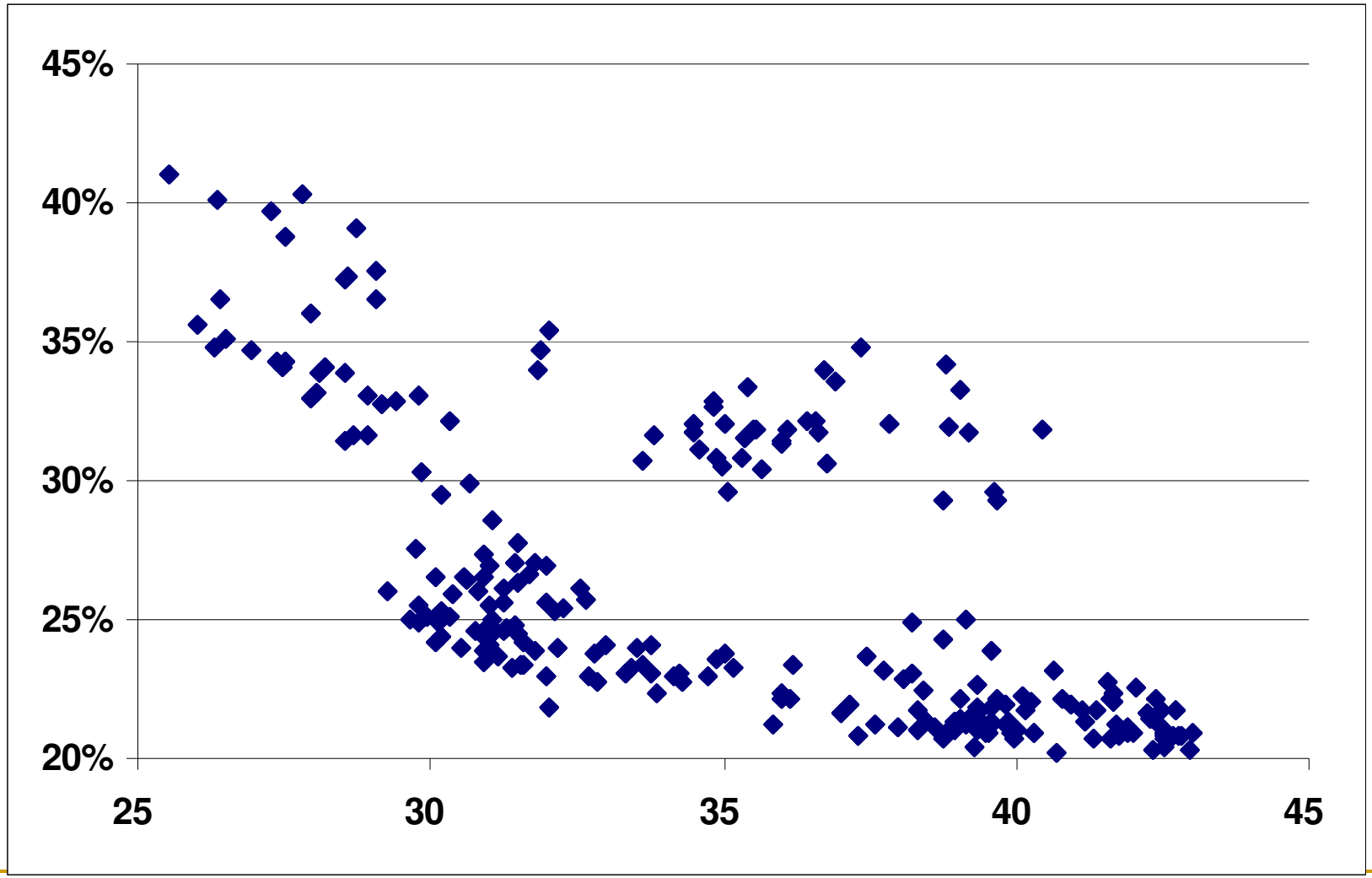
DUK



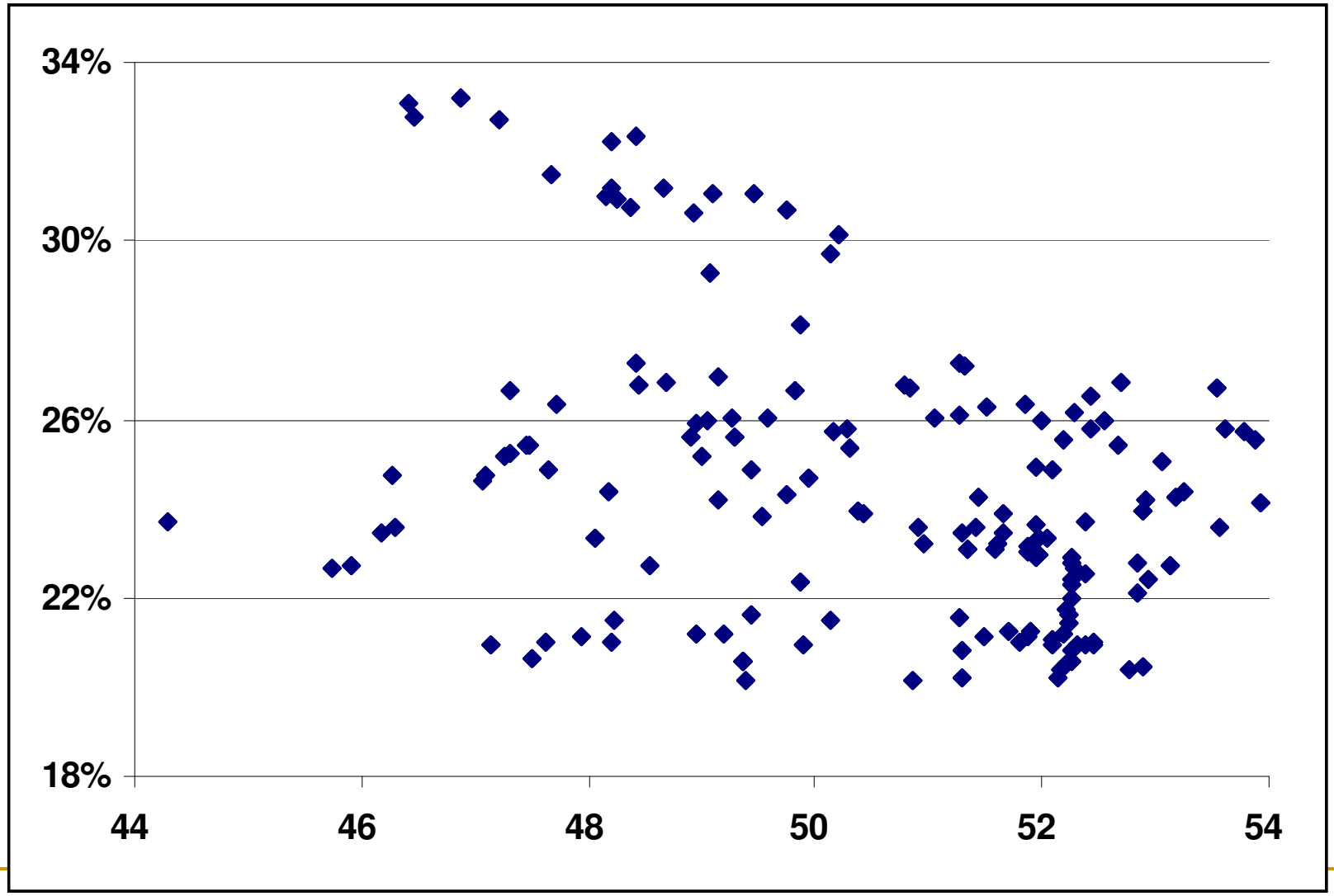
IPG



CSX



SEE



The GM Saga

The GM Saga

- 16 March: GM announces losses for Q1
- 15 April: Q1 report, 10 year low, Kerkorian starts buying
- 4 May: Offer by Kerkorian at $31
- 5 May: Two notch downgrade of GM's debt by S&P

The GM Saga


GENERAL MOTORS
as of 3-Jun-2005
Splits:
36
34
32
30
28
26
24
Mar23
Apr12
May02
May20
80
60
40
20
0
Millions
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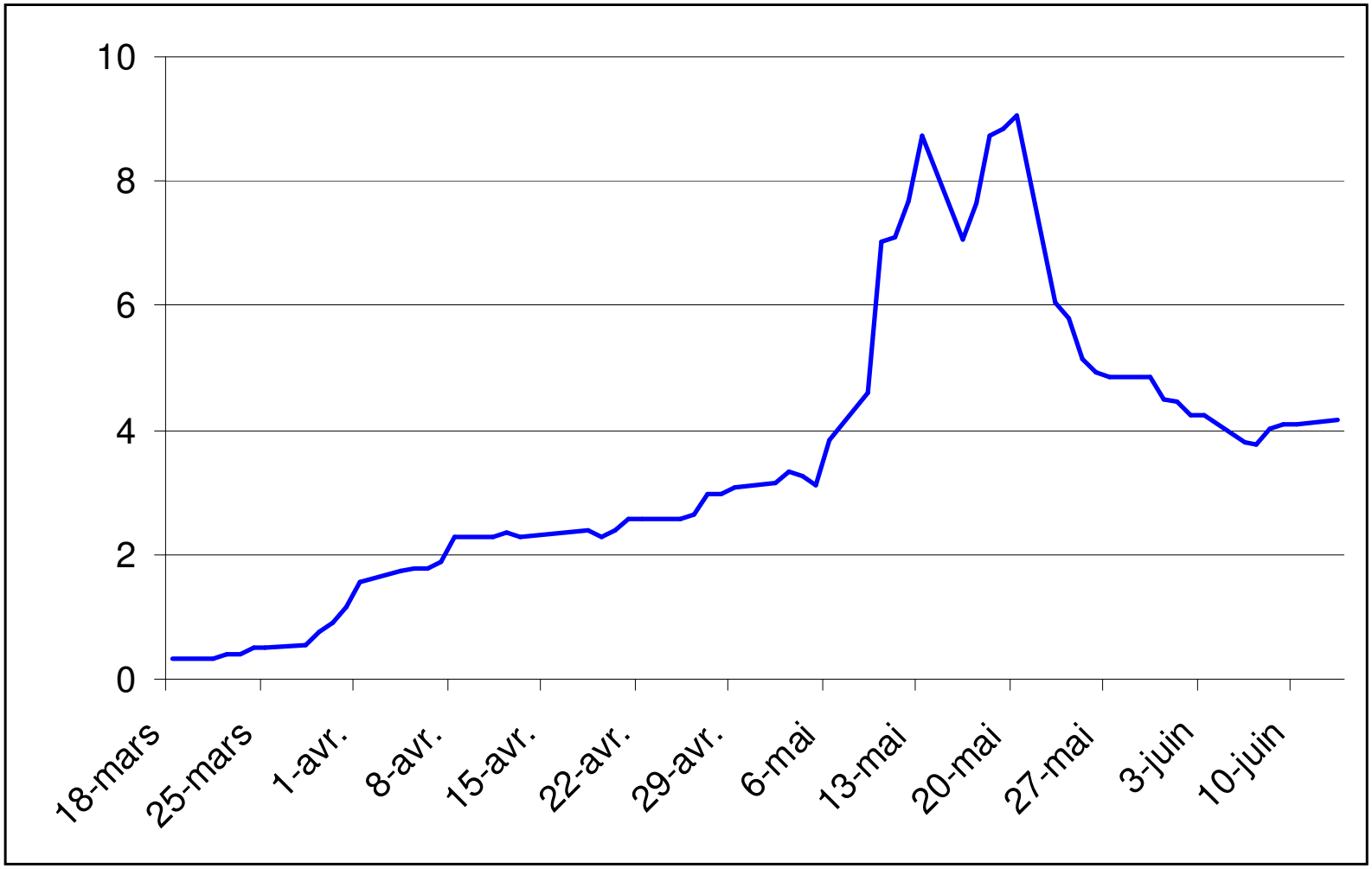
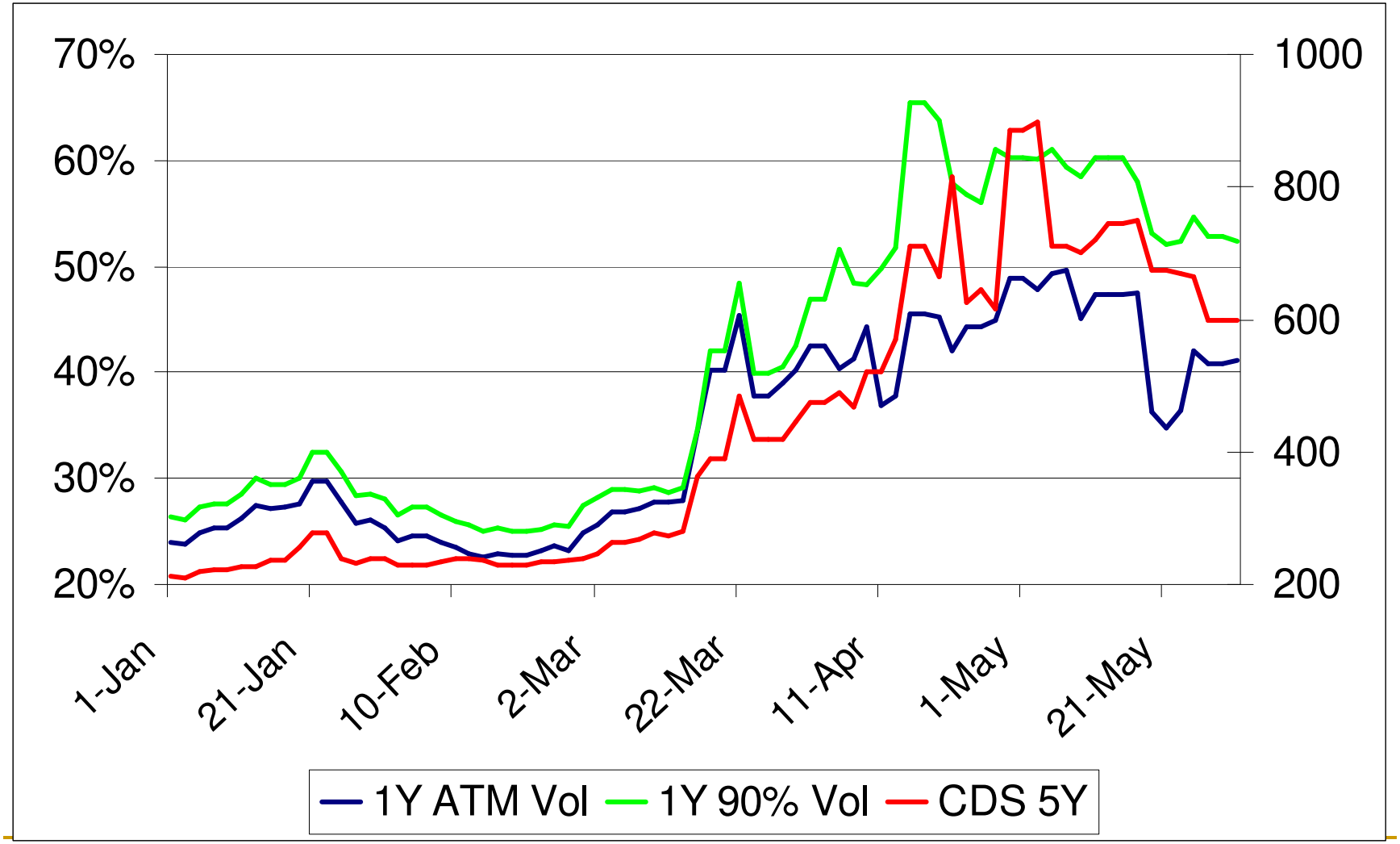
http://finance.yahoo.com/

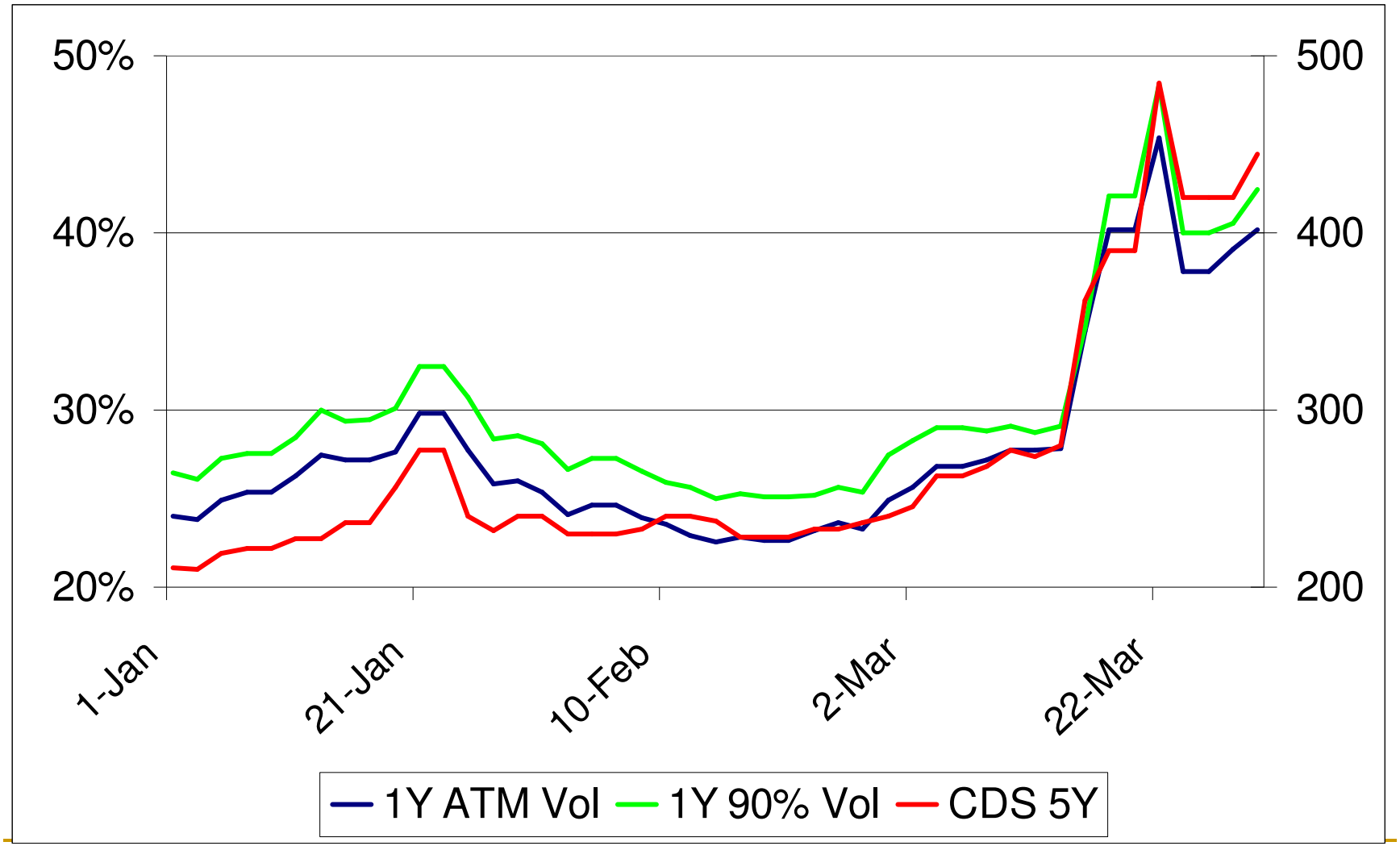
The GM Saga Stock Borrow



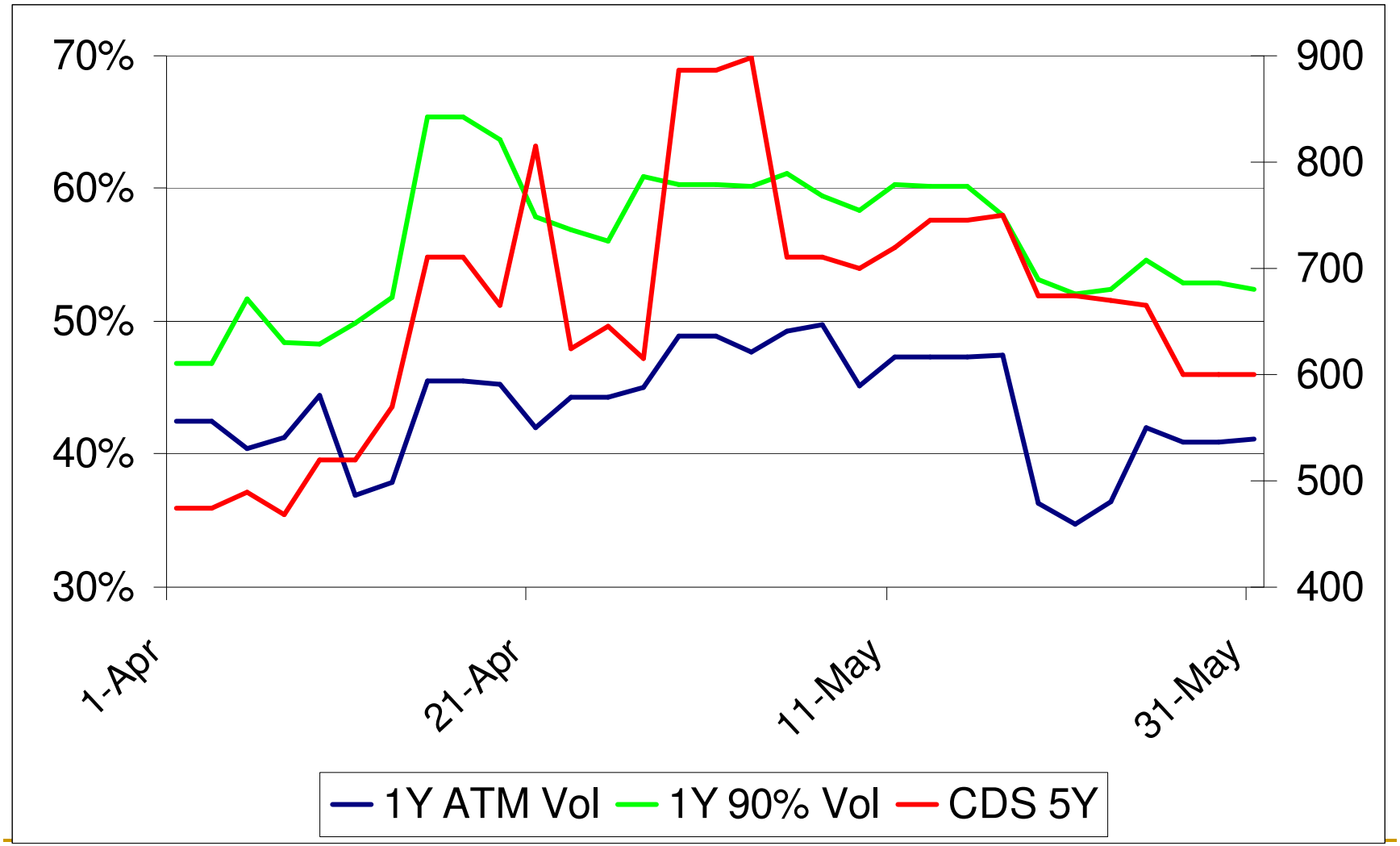
The GM Saga


70%
60%
50%
40%
30%
20%
1000
800
600
400
200
1-Jan
21-Jan
10-Feb
2-Mar
22-Mar
11-Apr
1-May
21-May
1Y ATM Vol
1Y 90% Vol
CDS 5Y

The GM Saga - up to 1 April 2005



The GM Saga - from 1 April 2005



GM 14 March 2005

	Implied Vol		
	90%	100%	110%
14-Jun-05	30.9%	26.8%	24.9%
14-Sep-05	29.6%	27.2%	25.1%
14-Dec-05	29.3%	27.6%	26.3%
14-Mar-06	29.1%	27.8%	26.7%

CDS 5Y 280bp

Spot 34.3

	Regime 1	Regime 2
Total Vol	29.4%	28.8%
Brownian Vol	19.6%	15.8%
Default	240bp	577bp

From reg1 to reg2

Intensity 0.57

Jump -20%

GM 18 March 2005

	Implied Vol		
	90%	100%	110%
18-Jun-05	47.4%	43.2%	39.7%
18-Sep-05	45.9%	42.3%	38.9%
18-Dec-06	43.5%	40.8%	38.3%
18-Mar-06	42.1%	40.2%	38.7%

CDS 5Y 390bp

Spot 28.62

	Regime 1	Regime 2
Total Vol	33.4%	28.8%
Brownian Vol	33.4%	14.4%
Default	1203bp	497bp

From reg1 to reg2

Intensity 2.37

Jump 16%

GM 3 May 2005

	Implied Vol		
	90%	100%	110%
3-Aug-05	56.9%	51.0%	33.1%
3-Nov-05	57.5%	51.0%	33.2%
3-Feb-06	58.2%	49.7%	32.9%
3-May-06	60.2%	47.8%	33.1%

CDS 5Y	899bp
Spot	27.77
HVol 260	32.6

	Regime 1	Regime 2
Total Vol	35.3%	24.0%
Brownian Vol	0%	0%
Default	0.45	0.12

From reg1 to reg2	
Intensity	4.6
Jump	15%

GM 3 May 2005

- Convert GM 2033 @ 6.25%
- Bid – Ask: US$ 19.0 - 20.3
- Model price: US$ 19.3
- Delta 0.493

- Hedging with CDS 5Y
- Underlying: 0.489
- CDS 5Y: -9.55